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THE ARITHMETIC TEACHER

a journal of
The National Council of Teachers of Mathematics

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THE ARITHMETIC TEACHER

Volume V Number 3

April 1958

Research on Arithmetic Instruction—1957

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One year ago The Arithmetic Teacher contained a summary of research on arithmetic instruction published during the six calendar years, 1951 through 1956.[1] The present summary represents a continuation of the previous one and brings it up-to-date through the calendar year of 1957.

Two things have been done which, it is hoped, will increase the value of the 1957 summary. First, the annotations accompanying each reference have been made more informative than those in the 1951–56 summary. Second, a separate section of the present summary is devoted to unpublished doctoral theses and dissertations completed in connection with degrees granted during 1957. This is a new section, not included in the earlier six-year summary.

SECTION I: PUBLISHED RESEARCH

The annotated references in this section are limited to *published* articles, monographs and the like. A serious attempt was made to make the listing a relatively complete one. However, a limited degree of selectivity did govern the compilation of references.

The listing of 25 research reports in this section includes two (Nos. 11 and 14) that were published in 1956 but came to the writer's attention too late for inclusion in the previous six-year summary. These references have been starred in the annotated bibliography.

In the listing which follows, the writer applied the same criteria of delimitation that were used in the 1951–56 summary. Thus, published references included in the bibliography are restricted to: (1) normative and experimental studies which report specific data or findings on a problem associated with, or related to, mathematics instruction at the elementary-school level; and (2) bibliographies, summaries, and more or less critical discussions which relate, in whole or major part, to such normative and experimental studies.

Research Listing for 1957

1. Aftreth, Orville B. "Shall We Expose Our Pupils to Errors?" The Arithmetic Teacher 3: 129–31; April 1957.

Reports on, and summarizes findings from, the author's doctoral dissertation, "The Effect of the Systematic Analysis of Errors on Achievement in the Study of Fractions at the Sixth Grade Level." Control and experimental groups were formed from 289 pupils in seven sixth-grade classes. Over a three-month period, children in the control groups received instruction in addition and subtraction of common fractions and worked 19 sets of practice exercises, each having 20 examples. During the same period, children in the experimental groups received comparable instruction but their practice took the form of identifying and correcting errors in 19 sets

[1] Weaver, J. Fred. "Six Years of Research on Arithmetic Instruction: 1951–1956." The Arithmetic Teacher 4: 89–99; April 1957.

of exercises, each having 20 examples with answers. Experimental hypotheses were tested in terms of the criterion variable based on both an immediate recall test and a delayed recall test. Findings prompted the conclusion that exposing pupils to errors in addition and subtraction of common fractions did not affect their learning of these operations adversely.

2. ANDERSON, GEORGE R. "Visual-Tactual Devices and Their Efficacy." THE ARITHMETIC TEACHER 4: 196–203; November 1957.

Reports a study ". . . to learn what effect, if any, a kit of sixteen visual-tactual devices had upon the learning of a unit involving areas, volumes, and the Pythagorean relationship in eighth grade arithmetic." Five hundred forty-one students from three suburban junior high school eighth grades were distributed among 18 sections—nine of which served as the experimental group (using the kit of visual-tactual devices), and nine of which served as the control group (not using the kit of visual-tactual devices). Pupils were tested on relevant experimental factors prior to the eight-week instructional period, were tested on two occasions during the period (at the end of $3\frac{1}{2}$ and 8 weeks), and were tested 12 weeks after the end of the instructional period. Findings and conclusions were based on 105 matched pairs of girls and 99 matched pairs of boys. The experimental sections scored higher than the control sections on both the criterion and retention tests, but none of the differences was statistically significant. It further was found that among pupils of lowest MA, those in the control sections scored higher than those in the experimental sections, but not significantly so. Use of visual-tactual materials was found to have no significant effect on attitude toward arithmetic. Also, the extent to which pupils used visual-tactual materials was found not to be correlated significantly with scores on the criterion test. There was no significant difference between the findings for boys and those for girls.

3. CURTIN, JAMES. "Arithmetic in the Total School Program." THE ARITHMETIC TEACHER 4: 235–39; December 1957.

A section of this article summarizes some of the research studies devoted to the "Frequency of Arithmetical Terms in Texts and References in Other Fields in the Elementary School Curriculum."

4. EADS, LAURA K. "Learning Principles that Characterize Developmental Mathematics." THE ARITHMETIC TEACHER 4: 179–82; October 1957.

The latter portion of this article briefly describes some of the procedures and techniques used in New York City's long-range program of curriculum or "action" research relating to Developmental Mathematics.

5. FLOURNOY, FRANCES. "A Consideration of the Ways Children Think When Performing Higher-Decade Addition." *The Elementary School Journal* 57: 204–208; January 1957.

Reports a study of the thinking patterns used by, and taught to, children when working with higher-decade addition examples. Data were derived from the performance of 48 third-grade pupils on three specially constructed tests, designed to reveal the thinking patterns actually used by the children, and from interviews with the teachers of the pupils tested. The researcher infers that children use "carrying" more often than "adding by endings" in examples such as $48+5$ because many teachers do not understand the thought pattern involved in the latter procedure (considered by the researcher to be the procedure most commonly advocated in the professional literature).

6. FLOURNOY, FRANCES. "Developing Ability in Mental Arithmetic." THE ARITHMETIC TEACHER 4: 147–50; October 1957.

Reports an analysis of six fifth-grade arithmetic textbooks, published within the past five years, in regard to the kinds and amount of mental arithmetic exercises included. Cites the need for improvement in textbooks and in instructional practices and emphases along the line of mental arithmetic.

7. GIBB, E. GLENADINE, and H. VAN ENGEN. "Mathematics in the Elementary Grades." *Review of Educational Research* 27: 329–42; October 1957.

Summarizes 112 research studies and re-

lated references on the teaching of mathematics in the elementary-school. Discusses these in relation to one or another of 17 categories (e.g., Methods of Instruction, Learning, Operations, Arithmetic Textbooks, and the like). Covers the six-year period since the similar summary in the October 1951 issue of the *Review* devoted to the broad field of "The Natural Sciences and Mathematics." (See also reference No. 15 in this present article.)

8. GUNDERSON, AGNES G., and ETHEL GUNDERSON. "Fraction Concepts Held by Young Children." THE ARITHMETIC TEACHER 4: 168–73; October 1957.

Reports findings from interviews with 22 second-grade children following an introductory oral-manipulative lesson dealing with halves and fourths. Emphasizes the need for planned, systematic instruction with fraction concepts at the second-grade level, and the ability of children at this grade level to comprehend and profit from such instruction.

9. HARTUNG, MAURICE L. "Estimating the Quotient in Division." THE ARITHMETIC TEACHER 4: 100–111; April 1957.

Presents a critical analysis of research relating to methods of estimating quotient digits when dividing by two-figure divisors. Discusses the relative merits of three methods of estimation: (1) the "round down" (or "apparent" or "one-rule") method, (2) the "round both ways" (or "increase-by-one" or "two-rule") method, and (3) the "round up" method, which has received relatively little instructional attention until recently. Presents and interprets selected statistical analyses in scholarly terms, using rigorous symbolic notation that may be more confusing than helpful to the mathematically naive reader. Points to the difficulties encountered in attempting to set up an appropriate and adequate experimental design for a definitive study involving experimentation with children. Recommends adoption of the "round up" method on the basis of his analysis and interpretation of existing data and principles of learning theory.

10. HARTUNG, MAURICE L. "Selected References on Elementary-School Instruction: Arithmetic." *The Elementary School Journal* 58: 170–72; December 1957.

This year's edition of this annual *Journal* listing includes 22 selected references, with brief accompanying annotations, relating to various phases of arithmetic instruction in the elementary-school. Some, but not all, of the references pertain to research studies.

*11. HOLMES, DARRELL, and LOIS HARVEY. "An Evaluation of Two Methods of Grouping." *Educational Research Bulletin* 35: 213–22; November 1956.

Compares the relative merits of "permanent" vs. "flexible" grouping of children within a class for arithmetic instruction, based on experimental studies conducted in two third-grade classes, two fourth-grade classes, and two sixth-grade classes. Data relating to effects on arithmetic achievement were collected for all classes. Additional data were collected from the sixth-grade classes in relation to the effects on attitudes toward arithmetic and on the social structure of the classes. Concludes that ". . . the method used in grouping arithmetic classes in order to meet individual differences is not particularly crucial."

12. HOWARD, CHARLES F. "British Teachers' Reactions to the Cuisenaire-Gattegno Materials." THE ARITHMETIC TEACHER 4: 191–95; November 1957.

Reports a study of the Cuisenaire-Gattegno color-rod approach to the teaching of arithmetic, based on observations in 22 infant- and junior-school classes in the London (England) area, and on interviews with 31 teachers, as well as on classroom demonstrations by Dr. Gattegno. Summarizes teachers' opinions on 17 questions related to the materials and their use. Concludes that: (1) the Cuisenaire-Gattegno color-rod approach is valuable and holds promise for further development; (2) although slower learners benefit to some extent from the color-rod approach, the average and brighter children seemed to benefit to a greater extent; (3) ". . . certain mathematical concepts that are not usually developed easily in children by current approaches to arithmetic were facilitated considerably by the use of the material

in the recommended manner; and (4) "At present the Cuisenaire-Gattegno approach holds considerable promise as a supplement to current methods, and further studies should be made to evaluate its effectiveness and to develop the procedures."

13. IVIE, CLAUDE, LILYBEL GUNN, and IVON HOLLADAY. "Grouping in Arithmetic in the Normal Classroom." THE ARITHMETIC TEACHER 4: 219–21; November 1957.

Reports briefly on some phases of an "action" or curriculum research study, with special emphasis upon a policy for meeting individual differences. The reader should not overlook the significant Editor's Note at the end of the report.

*14. LANKFORD, FRANCIS G., JR., and EVAN G. PATTISHALL, JR. *Development of Independence in Adding and Subtracting Fractions.* Charlottesville: University of Virginia Council for Educational Research, 1956. 69 p.

Reports on a pilot study and subsequent controlled experimental study relating to an evaluation of instructional procedures which encourage independence and flexibility of thinking and performance. Data for the latter study were derived from 18 control and 18 experimental fifth-grade classes learning to add and subtract common fractions during a period of approximately four months. Experimental classes used procedures and specially prepared instructional materials which encouraged and emphasized exploration, independence and flexibility of both thinking and performance. Control classes were taught in the more or less "usual" manner with emphasis upon relatively standardized or stereotyped ways of thinking and performing. Final evaluation was made on the basis of four tests and embraced factors such as: skill in addition and subtraction with common fractions, extent of need for written algorisms, conceptual learnings associated with the processes involved, transfer of learning to an untaught process (dividing whole numbers by common fractions), and pupil-interest. Experimental classes were found to be significantly superior (at the 5%-level) to the control classes in skill on the computation test and in ability to express arithmetical concepts through mental (nonwritten) computation, and were found to be slightly superior, but not significantly so, in their ability to deal independently with the untaught process. Pupils in experimental classes needed to rely on written forms of computation only half as much or often as did pupils in the control classes; also, the experimental pupils expressed a greater degree of interest in opportunities to compute mentally without pencil and paper. Among other things the researchers conclude that: "Perhaps the most important fact demonstrated by this study is that it is a sound procedure to allow pupils to use as much freedom and exploration as they require to understand fully working with fractions."

(NOTE—The following "non-research" article may be of interest to the reader in relation to the instructional approach emphasized in the above research study: Weaver, J. Fred. "Developing Flexibility of Thinking and Performance." THE ARITHMETIC TEACHER 4: 184–88; October 1957.)

15. MESERVE, BRUCE E., and JOHN A. SCHUMAKER. "The Education of Elementary-School Teachers." *Review of Educational Research* 27: 381–84; October 1957.

Summarizes nine studies pertaining to the education of elementary-school teachers, particularly in relation to their preparation in appropriate background mathematical content. Covers the six-year period since the similar summary in the October 1951 issue of the *Review* devoted to the broad field of "The Natural Sciences and Mathematics." (See also reference No. 7 in this present article.)

16. MILLER, G. H. "How Effective Is the Meaning Method?" THE ARITHMETIC TEACHER 4: 45–49; March 1957.

Reports a study of the "Rule Method" vs. the "Meaning Method" of arithmetic instruction at the seventh-grade level. Based on data collected from 180 matched pairs of pupils tested at the beginning and end of a semester with the California Arithmetic Test and a specially constructed "Meaning

Test," and on 95 of the 180 matched pairs tested again after the summer vacation. In general, concluded that the "Meaning Method" was more effective in relation to factors such as: computational skill, understanding of arithmetic principles, and comprehension of complex analysis. Also concluded that the "Meaning Method" was more effective for the average and the high IQ groups, whereas the "Rule Method" was more effective for the low IQ group. These latter conclusions were open to some question, however, because of the operation of a bilingual factor. Recommended continued research, with improvements in experimental design, relating to the effectiveness of the "Meaning Method."

17. PIKAL, FRANCES. "Review of Research Related to the Teaching of Arithmetic in the Upper Elementary Grades." *School Science and Mathematics* 57: 41–47; January 1957.

Reviews major findings from 12 selected research studies published since 1940, and points to their implications for the teaching of arithmetic particularly at the fourth-grade level. Classifies and discusses the studies in relation to the following categories: research on methods of teaching, on arithmetic concepts learned and used outside of school, on the subtraction process, on the division process, on the relationship of reading to achievement in problem solving, and on remedial arithmetic.

18. PRIORE, ANGELA. "Achievement by Pupils Entering the First Grade." THE ARITHMETIC TEACHER 4: 55–60; March 1957.

Describes the nature of an oral inventory test of number abilities administered individually to 70 children at the beginning of their first-grade work. Reports findings of observed abilities relating to rote and rational counting, identification and reproduction of groups, combining and separating groups, recognition of number symbols, recognition of fractional parts, and aspects of measurement, United States money, and time. Concludes that an incidental approach to arithmetic instruction in Grade I would be inadequate and unsatisfactory. Cites the definite need for, and value in, a systematic program of instruction in arithmetic at the first-grade level, based on children's existing quantitative knowledge and ability, and emphasizing appropriate understandings or meanings.

19. SCHOTT, ANDREW F. "New Tools, Methods for Their Use, and a New Curriculum in Arithmetic." THE ARITHMETIC TEACHER 4: 204–209; November 1957.

Discusses the development of special instructional materials, methods and sequences for the teaching of arithmetic, including the use of commercial calculating machines in their various forms. Reports experimental data relating to the achievement of children in first-, second-, and third-grade classes over a two-year period in an attempt to substantiate the following contention: ". . . it can be concluded that the new tools and methods for their use in a new curriculum have improved the learning of arithmetic in grades 1–3 as measured by the California Primary and Elementary Arithmetic Tests." *Also, very importantly, sounds the following note of caution:* "The research in the use of calculating machines to date is not yet complete enough to be conclusive. There is grave danger that schools will, in their desperation to improve their arithmetic programs, spend large sums of money for these machines without waiting for sound evidence as to the value of such tools of learning."

20. SCURRAH, MARK B. "Research in Arithmetic." Albany: Bureau of Elementary Curriculum Development, State Education Department, 1957. 15 p. (mimeographed)

Discusses findings and implications of research on various phases of arithmetic instruction, based on a bibliography of 55 references.

21. STIPANOWICH, JOSEPH. "The Mathematical Training of Prospective Elementary-School Teachers." THE ARITHMETIC TEACHER 4: 240–48; December 1957.

Reviews previous research and reports new research (based on a questionnaire study involving a jury of 70 "carefully se-

lected mathematics-education specialists) relating to the pre-service mathematical preparation of elementary school teachers. Makes six recommendations covering various aspects of the problem.

22. ULRICH, LOUIS E., SR. "100% Automatic Response?" THE ARITHMETIC TEACHER 4: 161–67; October 1957.

Reports a study of success and error with 2-figure multipliers among 32 children in one Grade 5B class. Data were derived from the administration of 15 tests, each having 10 examples of the most complex "process pattern" (Type 48), according to the author's elaborate analytical scheme (see Ulrich's *Streamlining Arithmetic*, Lyons and Carnahan, 1943). Concludes that ". . . it is very difficult to get 100% automatic response even when the possible variables are reduced to a minimum," that "process patterns are not few and simple but become quite numerous and complex but show relationships that make example classification possible," and that "children's errors are not always due to inability to work examples, but to various other psychological factors. . . ."

23. UNIVERSITY OF ILLINOIS COMMITTEE ON SCHOOL MATHEMATICS PROJECT STAFF. "Arithmetic with Frames." THE ARITHMETIC TEACHER 3: 119–24; April 1957.

This is not a research article *per se*. Rather, it is the outgrowth of significant on-going research. The article forwards implications and suggestions for mathematics instruction at the elementary-school level, based on extensive experimental work during the past several years at the high school level: the Project for the Improvement of School Mathematics, directed by the University of Illinois Committee on School Mathematics. Presently the Committee's experimental program is being extended downward to both the junior high school and the elementary-school levels. Teachers need to be familiar with this article, and need to be on the lookout for future articles by the Project Staff that may relate to mathematics instruction at the elementary-school level.

24. WANDT, EDWIN, and GERALD W. BROWN. "Non-Occupational Uses of Mathematics." THE ARITHMETIC TEACHER 4: 151–54; October 1957.

Reports a study of the non-occupational uses of mathematics (mental and written, approximate and exact) as recorded during a 24-hour period by 147 students enrolled in the authors' classes. Three-fourths of the reported uses were found to be mental rather than written, and 31% of the reported uses were approximate rather than exact. Seven-eighths of the approximate uses were also mental in contrast with written uses. The authors conclude that ". . . considerable emphasis should be placed on mental and approximate mathematics at both elementary and secondary levels."

25. WEAVER, J. FRED. "Six Years of Research on Arithmetic Instruction: 1951-1956." THE ARITHMETIC TEACHER 4: 89–99; April 1957.

Lists and briefly characterizes 71 reports of research on arithmetic published during the calendar years 1951–56. Points to the following areas of greatest research emphasis in terms of published reports: (1) mathematical background and preparation of elementary-school teachers, and (2) problem solving and patterns of quantitative thinking and performance. Cites a need for more sustained research interest and effort on the part of individuals, for an identification of the most crucial issues and problems needful of investigation, and for an increased coordination of attack upon these from various sources.

Sources of Research Reports

In what professional sources were research reports found most commonly? It is quite obvious that THE ARITHMETIC TEACHER has been by far the major source of research reports on arithmetic instruction published during 1957. Seventeen of the 25 reports listed—or 17 of the 23 reports published in 1957—appeared in this journal.

A year ago the writer stated: "We have good cause to be pleased with the research emphasis found in THE ARITHMETIC

TEACHER."[2] During the past year this emphasis actually has increased.

Types of Research Reports

In general, what major types of reported research[3] were published during the 1957 calendar year? As in the 1951–56 summary, a three-fold classification was used in an attempt to answer this question: (1) reports giving emphasis to data, findings, implications and the like from *experimental* studies; (2) reports giving emphasis to data, findings, implications and the like from *normative* or *status* studies; and (3) reports which are research summaries, critical discussions, annotated listings, etc., or which do not fall reasonably in either category (1) or (2).

On the basis of the writer's personal judgment, the tabulation below indicates the number of citations classified in each of the three categories.

Reports of experimental studies	8*
Reports of normative studies	9
Other types of reports	8

Areas of Research Emphasis

What aspects or phases of arithmetic instruction have received greatest research attention or emphasis? The answer to this question can be inferred from the following tabulation. Each published research report has been classified in one or another of the categories listed at the left below. For each category, the numbers at the right are *not frequencies;* rather, they refer to specific research reports in the basic listing.

Common Fractions	8, 14
Curriculum Development	4, 13
Division with 2-digit Divisors	9
Exposure to Error	1
General Research Summaries	7, 10, 17, 20, 25
Higher-decade Addition	5
Instructional Materials and Related Methods	2, 12, 19
"Meaning Method"	16
Mental Arithmetic	6
"Modern Mathematics" in the Elementary School	23
Multiplication with 2-figure Multipliers	22
Number Knowledge of School Entrants	18
Teacher Education	15, 21
Uses of Mathematics	3, 24
Provision for Individual Differences	11

Obviously, some references could have been classified above in more than one category. However, it was decided arbitrarily not to use such a procedure. The reader should not overlook the fact, nevertheless, that numerous studies in the basic listing deal with "instructional method" in one way or another, although a special category has not been set up for classification on this basis. (A good example of such a study would be No. 14.)

Sustained Research Effort

To what extent have persons who contributed to the published research during 1951–56 continued to contribute research reports to our professional literature during 1957? On the basis of the writer's previous summary coupled with the two reports from the current summary that actually were published in 1956, sixty-seven different persons were responsible for the published research reports summarized for the six calendar years, 1951–1956. Eight of these 67 persons were among the contributors to the published research reports listed in the 1957 summary. However, only three of these eight persons actually reported more or less new normative or experimental research.

SECTION II: UNPUBLISHED DOCTORAL THESES AND DISSERTATIONS

Graduate students' theses and dissertations represent one of our more important sources of research on arithmetic instruction. Unfortunately, much of this work never appears in published form. It thus escapes the attention of all too many persons. It is hoped that this section of the current summary may help in some way to remedy this situation.

The studies listed and annotated below represent unpublished *doctoral* theses and

[3] Weaver, *op. cit.*, p. 97.

* Two of these eight were the 1956 publications included in the present listing for reasons indicated previously.

dissertations completed in connection with *degrees awarded in the 1957 calendar year.* Masters' theses have *not* been included—largely because of the magnitude of the task of compiling a reliable listing of such studies. The bibliography of doctoral research on arithmetic was compiled from two sources: (1) returns on a questionaire sent by the writer to graduate schools or colleges of education, and the like, and (2) *Dissertation Abstracts*, Volume 17.[3]

The reader will find an incompleteness in some phases of the following listing. This is due largely to the paucity of information that could be secured on some of the studies, —at least within the time available to the writer. The annotations admittedly are brief and touch only upon selected findings and conclusions. In any event, it is hoped that the listing will serve a useful purpose to some extent.

Research Listing for 1957

1. BARTRUM, CHESTER. "An Analysis and Synthesis of Research Relating to Selected Areas in the Teaching of Arithmetic." (Ph.D., Ohio State University, 1957. Lowry W. Harding, Faculty Advisor.)

2. BROWN, BETTY IRENE. "A Study in Mental Arithmetic: Proficiency and Thought Processes of Pupils Solving Subtraction Examples." (Ph.D., University of Pittsburgh, 1957. Herbert T. Olander, Faculty Advisor.)

Found differences in ability to work subtraction examples mentally and with pencil-and-paper among children in grades 6–12. Concluded that greater attention should be given to mental arithmetic for all children, and particularly for highly intelligent and gifted ones.

(Also see *Dissertation Abstracts* 17: 2219; October 1957.)

3. CARPENTER, CLAYTON LEE. "Identification and Measurement of Arithmetical Concepts and Abilities of Kindergarten, First and Second Grade Children." (Ed.D., University of Nebraska Teachers College, 1957. Charles O. Neidt, Faculty Advisor.)

Reported differences in achievement between boys and girls, between urban and rural children, etc. Emphasized the need for more attention to the development of number *concepts* in the primary grades.

(Also see *Dissertation Abstracts* 17: 2205–2206; October 1957.)

4. BJONERUD, CORWIN. "A Study of the Arithmetic Concepts Possessed by Children on Admission to Kindergarten." (Ed.D., Wayne State University, 1957. Charlotte Junge, Faculty Advisor.)

5. COSGROVE, GAIL EDMUND. The Effect on Sixth-Grade Pypils' Skill in Compound Subtraction When They Experience a New Procedure for Performing This Skill." (Ed.D., Boston University School of Education, 1957. J. Fred Weaver, Faculty Advisor.)

Found no lasting interference effects, even among lower IQ pupils, when children who were habitual users of the decomposition method learned and practiced the equal additions procedure.

(Also see *Dissertation Abstracts* 17: 2933–2934; December 1957.)

6. DAUGHERTY, JAMES LEWIS. "A Study of Achievements in Sixth-Grade Arithmetic in Des Moines Public Schools." (Ed.D., Colorado State College, 1957. Paul McKee, Faculty Advisor.)

7. EIDSON, WILLIAM P. "The Role of Instructional Aids in Arithmetic Instruction." (Ph.D., Ohio State University, 1957. Lowry W. Harding, Faculty Advisor.)

8. HAGELBERG, RAYMOND RICHARD. "A Study of the Effectiveness of Specific Procedures for Solving Verbal Arithmetic Problems." (Ph.D., State University of Iowa, 1957. Herbert F. Spitzer, Faculty Advisor.)

Found that experience with special specific problem solving procedures was no more effective at the sixth-grade level than the instruction of the regular arithmetic program when measured by a standardized achievement test, but was more effective whe

[3] Published by University Microfilms, Ann Arbor, Michigan. Volume 17 includes the monthly issues for the 1957 calendar year.

measured by a special test adapted for the study.

(Also see *Dissertation Abstracts* 17: 2878–2879; December 1957.)

9. HIMMLER, MERWIN LEWIS. "An Analysis and Evaluation of a Television Demonstration of the Teaching of Fifth Grade Reading, Arithmetic, and French." (Ed.D., University of Pittsburgh, 1957.)

Found little observable difference between the effectiveness of TV teaching and regular classroom teaching, with reading more suitable than arithmetic or French for TV instruction.

(See also *Dissertation Abstracts* 17: 2467–2468; November 1957.)

10. JONES, EMILY K. "An Historical Survey of the Developmental Treatment of Vulgar Fractions in American Arithmetics from 1719 to 1839." (Ph.D., University of Pittsburgh. 1957. Herbert T. Olander, Faculty Advisor.)

11. KRAMER, KLAAS. "A Comparison of Objectives, Methods, and Achievement in Arithmetic in the United States and in The Netherlands." (Ph.D., State University of Iowa, 1957. A. N. Hieronymus and C. N. Singleton, Faculty Co-Advisors.)

In addition to various instructional differences, found Dutch pupils in Grades 5 and 6 to be significantly superior to their American counterparts on tests of both Arithmetic Problem Solving and Arithmetic Concepts. Forwarded reasons to account for these differences.

(Also see *Dissertation Abstracts* 17: 2881; December 1957.)

12. LOWRY, WILLIAM C. "The Implication of the Theory of Operationism and of Some Studies in Psychology and Anthropology for the Teaching of Arithmetic." (Ph.D., Ohio State University, 1957. Nathan Lazar, Faculty Advisor.)

13. MAURO, CARL. "A Survey of the Presentation of Certain Topics in Ten Series of Arithmetic Textbooks." (Ed.D., University of Maryland, 1957. Alvin W. Schindler, Faculty Advisor.)

Found considerable agreement among series *re* the grade-placement of initial systematic study of skills, found less agreement among series *re* the sequence of presentation of sub-skills, and found greatest differences among series *re* the method of introducing and presenting new sub-skills. Forwards reasons to account for such differences.

(Also see *Dissertation Abstracts* 17: 1515–1516; July 1957.)

14. RAPPAPORT, DAVID. "An Investigation of the Degree of Understanding of Meanings in Arithmetic of Pupils in Selected Elementary Schools." (Ph.D., Northwestern University, 1957. E. T. McSwain, Faculty Advisor.)

15. RUSCH, CARROLL ERNEST. "An Analysis of Arithmetic Achievement in Grades Four, Six, and Eight." (Ph.D., University of Wisconsin, 1957. A. S. Barr, Faculty Advisor.)

Based on a factor analysis of the "number factor," found the following sub-factors at each of the three grades for both boys and girls: abstraction sub-factor, analysis sub-factor, and application sub-factor. Considered Grade 6 to be crucial because "strange changes" in factorial make-up took place at this level.

(Also see *Dissertation Abstracts* 17: 2217; October 1957.)

16. SOLE, DAVID. "The Use of Materials in the Teaching of Arithmetic." (Ph.D., Teachers College, Columbia University, 1957. Howard F. Fehr, Faculty Advisor.)

Found that the use of a variety of materials in teaching an arithmetic topic does not produce better results than the use of only one material, if both procedures are used for the same length of time.

(Also see *Dissertation Abstracts* 17: 1517–1518; July 1957.)

17. WOZENCRAFT, MARIAN. "An Analysis of the Relationship between Certain Arithmetic Abilities, Certain Reading Abilities, and Intelligence." (Ph.D., Western Reserve University, 1957. David P. Harry, Jr., Faculty Advisor.)

Note 1: Several dissertations were completed which dealt with mathematics instruction at the junior high school level

exclusively. These were not included in the above listing because no portion of these studies involved the instructional program below the seventh grade level.

Note 2: References not listed in Volume 17 of *Dissertation Abstracts* are likely to appear in Volume 18 (1958).

Summary

In Section II, more studies were devoted to an investigation of arithmetic achievement of one kind or another at various grade levels than were devoted to any other thing in general: Nos. 3, 4, 6, 11, 14 and 15.

Furthermore, almost half (47%) of the studies listed in Section II came from three institutions: Ohio State University (3 studies), University of Pittsburgh (3 studies), and the State University of Iowa (2 studies). The remaining nine studies (53%) came from nine different institutions.

The writer is encouraged by the amount of research that is appearing in published form. It is his personal feeling that THE ARITHMETIC TEACHER has done much to spur on this tendency. It is hoped that this trend will continue and even become accelerated.

Those who engage in research work should sense a definite obligation to do at least two things: (1) to report their research so that it may be of wider benefit than would be the case otherwise, and (2) to report their research in such a way that it actually tells the research story adequately. The overall quality of arithmetic instruction will be improved in direct proportion to the extent that research workers accept these obligations in good faith.

EDITOR'S NOTE. Again we are very grateful to Professor Weaver for his excellent compilation and annotation of research dealing with arithmetic instruction during the past calendar year. Advanced undergraduate students, graduate students, and their teachers as well as teachers and supervisors in the field will find this a very helpful listing.

What Does 6-10-57 mean?

To answer this question, it would be desirable to state that it means a date of the year 1957. Now we have to find out what date this is?

Before taking up this question, let us take a few examples from everyday life.

a. In writing an address on a letter, one goes from the name of the person to the number of the house, from the number of the house to the name of the street, then to the name of the town and then to that of the state. In other words from particular to general.

b. When naming a child, we first give him his name and then suffix the family name. Never has one seen the family name first and then the name of the child or person, except in a bibliography where we then need a comma to distinguish.

c. Our love is first for the nearest relation, then for family, then for the community, then for the state and then for the country and so on.

Our ideas of order demand that we arrange things in sequence whenever possible.

One other point for consideration is that we almost always know what year it is and usually the month. The day of the month is really the thing in which we are most frequently interested.

Thus it seems logical that while writing a date we should begin with the day, go to the month and last to the year.

Now let us see what date the figure 6-10-57 stands for. To most people it will appear the tenth day of June in the year 1957.

To me, it appears that 6-10-57 should mean 6th day of October, 1957 and not the 10th day of June, 1957. Isn't it logical and correct?

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The Concrete Devices of Structural Arithmetic

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IN THE TEACHING of science it is common practice to use models and devices which make it possible for the pupils to see with their own eyes all those usually "invisible" objects which they want to study. A model of the solar system, for instance, allows the pupils to discover the relative movements of earth, sun, and moon in a simple set-up which shows the structure of the system. Even abstract principles and formulas become clear when they are illustrated by experiments which the pupils carry out with structurally correct concrete devices.

STRUCTURAL ARITHMETIC, a new approach to the teaching of numbers, also uses models and devices to allow pupils to discover the structure of our number system and its interrelationships. The STRUCTURAL ARITHMETIC devices are structurally true to the number system, i.e., they show all the properties of abstract numbers. Moreover, the blocks which represent the numbers fit into specific receptacles in such a way that the children are able to *discover* all the number relationships which are to be studied and learned. In experiments of their own, the pupils discover addition and subtraction, and, later, multiplication and division, fractions, and so on. Since the number system shows unity and continuity, the same materials can be used in different experiments at higher levels. In this paper,[1] only some of the fundamental aspects of STRUCTURAL ARITHMETIC will be described, and only a few experiments can be shown in the short time limit.

In contrast to the usual introduction of numbers in today's schools, the STRUCTURAL ARITHMETIC method discards the *counting* of buttons, sticks, cats, or sheep to develop first number concepts.[2] Is it not true that if a child finds by counting buttons one-by-one that 6 buttons and 2 buttons are 8 buttons, he visualizes one disconnected number fact "$6+2=8$"? Does he grasp the principle that binds all facts of the $+2$ group together, i.e., that adding 2 means to reach the next higher even or odd number respectively? If we add 2 to 5, the result is 7. If we add 2 to 8, the result is 10 *and so on and so forth.* This "so on and so, forth" is not only one of the characteristics of arithmetic, but it is also an excellent illustration of how a few such "structural techniques," shown in unforgettable experiments and pictures, help the child to master 100 addition and 100 subtraction combinations which are usually taught separately and by drill.

The course of teaching STRUCTURAL ARITHMETIC starts with the Counting Board, the Pattern Boards, the Unit (Ten) Box, and the other Number Cases (boxes). The first experiments are number games without number names or symbols. The same devices lead to the use of number names in the games and then to number symbols. Let us look at the Counting Board and the Unit Box to demonstrate these three steps.

The Counting Board has ten grooves of different sizes. In the first experiment, the

[1] This paper is the basis of a talk at the 36th Annual Meeting at Cleveland, Ohio, of the National Council of Teachers of Mathematics.

[2] For a thorough discussion of Measuring versus Counting, see "Children Discover Arithmetic," Harper, 1949, p. 18 by the same author.

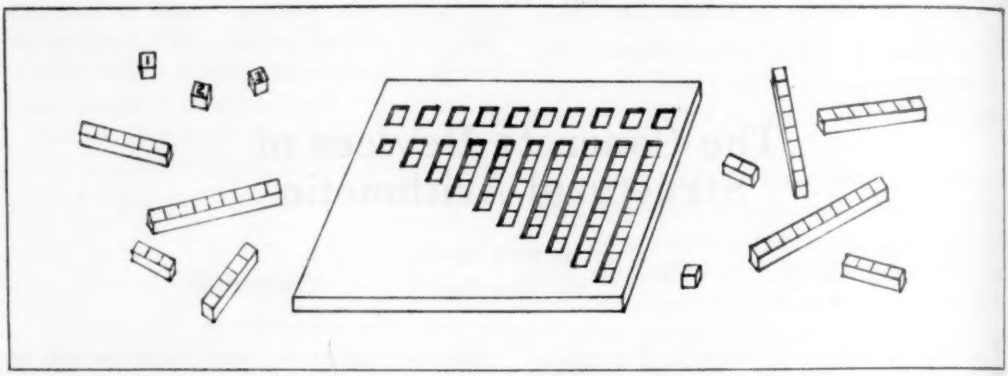

FIG. 1. The Counting Board.

child fits the corresponding blocks into the proper grooves. He notices that each block has its proper place in the sequence of blocks, and that the first block is very small, the next one is somewhat bigger, *and so forth,* to the last one that is the biggest. The game is self-corrective: only the proper blocks fit the proper grooves, and only if put into their proper places are the blocks "used up" when the game is finished.

Although in practice the next experiment centers on a new device, in this paper we shall show what other things the child can discover with this same Counting Board. Using the Counting Board blocks, he learns to count the number of units in each block. After the initial counting experiment, he can identify the blocks individually and give them immediately their proper names. Without any further counting, he calls the blocks the "one-Block," the "two-Block," and so on. The children comment on the sizes and places where each block "lives," as e.g., "the one-Block comes first," the "nine-Block is very big, almost as big as the ten-Block," or, "that must be the six-Block, it comes after the five-Block." Numbers, in mathematics, have order and size. We see how this fundamental concept is grasped by the child.

Again, at a later date, the Counting Board is equipped with a Number Guide which shows the symbols from 1 to 10 above the corresponding blocks. The child examines each symbol and says: "this goes with the 3-Block so it must be three; the *3* is small compared with the *8;* the *5* follows the *4;* the *7* lives between *6* and *8*." The symbols stand for the blocks and acquire their characteristics, and thus the child talks about the abstract numbers in full comprehension of their meaning.

The same three steps work wonders when applied to the child's experiments with the Unit (ten) Box. On the "wordless" level, the child simply tries to fill the box with matching block pairs. He puts any block into the box and looks for another block that will just fill the gap. If he chooses a a block that is too big, it will not have room; if he tries one that is too small, there is a gap left. Only certain "block combinations" fit. (See Figure 2, next page.)

As soon as the child is able to call the blocks by their names, the filling in of block pairs gives rise to the "Story of 10." The child accompanies his moves with statements as: "The 7 needs 3 to make 10," "8 and 2 are as big as 10," "10 and 0 is 10," etc. The climax is reached, however, when the child knows what symbols stand for the blocks and can use meaningfully the plus sign to stand for "and" and an equal sign to stand for "as big as," or, "is," or "make." At this level, the experiments may be "recorded" with individual markers on which symbols and signs are painted. So, e.g., the child expresses the above-mentioned block-discoveries by arranging the symbols to say:

$7+3=10 \qquad 8+2=10 \qquad 10+0=10$

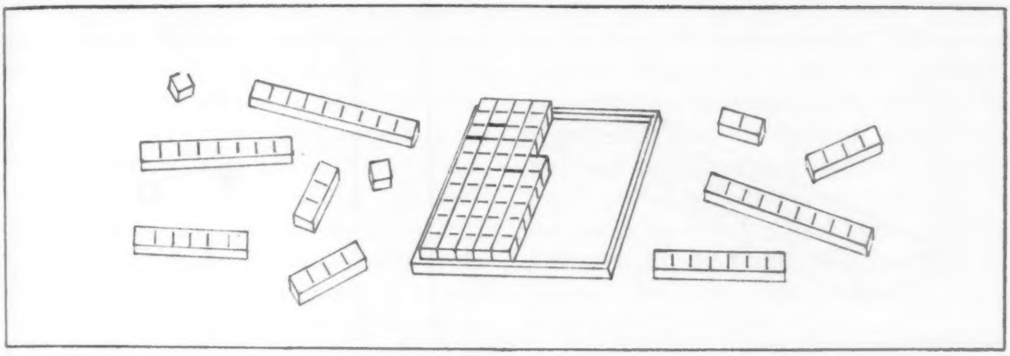

FIG. 2. The Unit Box.

Even the "zero-fact" is full of meaning since it says that the 10-Block fills the box by itself, that "nothing," i.e., no other block, would have room to "go with it."

From here on, all the number games which the child has played before are not completed for him until he has recorded his findings with the proper symbols and signs. Whether he plays with the Pattern Boards, the Unit Box, or the other Number Cases, the results of the experiments are expressed as "3+3=6," "5+2=7," or "9+0=9."[3]

Now, the teacher may say: "Put together 4 and 5 and tell me how big they are together." Accordingly, the child adds a 4-Block to a 5-Block and then finds a single block of the same total length; the 8-Block is too small, the 10-Block is too big. There is only one block that has the same length—the 9-Block. Four and five make *nine*, which he records as: "4+5=9." The teacher's question is answered with complete confidence. The teacher may also use the markers with the symbols to ask her question. If she set up "5+3=8," the child reads: five and three are as big as what other block? He puts the two blocks together and finds the result, "5+3=8." We see that the symbols and signs invite him to carry out the exact mathematical operation: the *plus* sign makes him *add* two blocks. Thus, the child can solve any addition example with sums not greater than 10 with the help of the blocks. If the teacher now gives the children real problems from daily life, the children are fascinated to see that they can find the answers with the materials. This stage is characterized by satisfying experimentation. No drill is used, and nobody forces the children to recall answers to addition facts. Full comprehension comes first; mastery will be attained at a later date.

The step to subtraction is an easy one once addition is understood. Again, there are many different experiments with various devices, which lead first to the understanding of subtraction facts, then to their expression with words and, finally, to their recording, as e.g. "10−4=6."

The teacher may introduce subtraction by standing up the numeral 10 above the 10-Block in the Counting Board. Now, the 10-Block is removed, and the child is asked to fill the groove with any two matching blocks, e.g. with the 6-Block and the 4-Block. The child states the familiar addition fact: 6 and 4 are just as big as 10. He sees with interest that the teacher removes the 4-Block, saying: Ten less four leaves what other block? The child looks at the "remaining" 6-Block, and gives the answer: the 6. The 4-Block is returned, the 6-Block is "taken away," and the pupil discovers: If I take 6 from 10, its friend (or: its partner), the 4-Block is left in the groove. As time goes on, the wording gets shorter and more exact, and, when the child experiments with other rows, he may find: 8 less 5

[3] The teacher finds all these steps in the Teacher's Manual for Use with Beginners, "Experimenting with Numbers," Houghton Mifflin Company, 1950 and 1954, by the author.

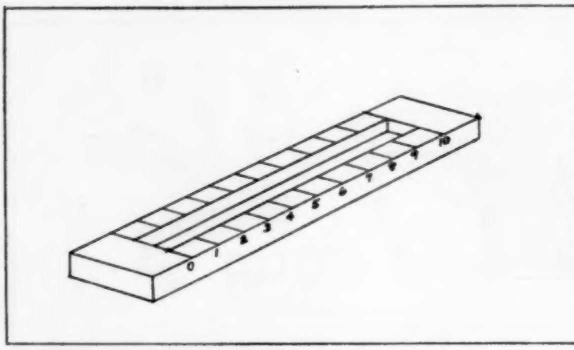

FIG. 3. The Number Track (1–10).

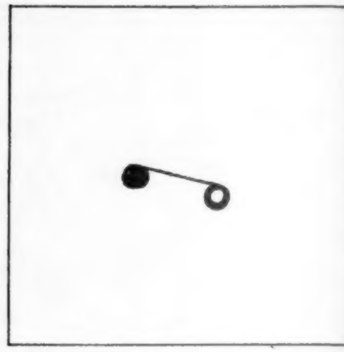

FIG. 4. A Jumper.

leaves 3, or 4 less 2 is 2, and so on. When the minus-sign is introduced to stand for the word "less," the child is ready to express his findings as:

$$10-6=4 \qquad 8-5=3 \qquad 4-2=2$$

The concept of subtraction is also presented in the Number Track, which is a device into which the 10-Block just fits (see Figure 3). It has numbers painted on it from 1 to 10 showing where the individual blocks reach. The 5-Block reaches the numeral 5, the 8-Block reaches the figure 8, and so on. First, addition facts are found in the Number Track. If we add 3 and 2, the combined block lengths reach the total 5. We actually don't have to use the blocks; instead we may use a little wire-bridge or "Jumper" which we may put on 3. (See Figure 4.) If we move this jumper "up 2," we arrive at 5, a new way to find the familiar fact: $3+2=5$. If we now put the Jumper on 5 and "go down 2" in the track, we reach 3, which discloses the fact: 5 less 2 is 3, or, $5-2=3$.

This last experiment shows with impressive clarity that addition and subtraction are opposite operations, one means "ascending" and the other "descending" the Number Track. One means *increasing* (putting blocks together), the other operation means *decreasing* (taking a block away or moving the Jumper down); if we add, we have more in the end, if we subtract we have less than at the start. If the teacher now writes on the blackboard: $1+8=$, the pupil follows the implied command by putting the 1-Block and the 8-Block together into the Number Track, finding that they equal 9 in all. Should the teacher write: $7-5=$, the pupil marks the 7 in the Number Track, then moves the Jumper down 5 places (the length of the 5-Block) to subtract 5 units, and sees that the result is 2.

The Mastery Stage

After the meaning of addition and subtraction is completely grasped, the pupil is ready to attain "mastery," i.e. he comes to the point where he must know the answers to the 55 addition facts and to the 55 subtraction facts in the range from 1 to 10. Here is one of the main differences between "learning by insight" in STRUCTURAL ARITHMETIC and the usual drill practices which try to fix the "combinations" in the child's memory. STRUCTURAL ARITHMETIC groups the facts to be learned in sets which are bound together by one easily grasped principle.

The "+1" and the "+2" facts (and the "−1" and the "−2" facts as well) are shown in "unforgettable pictures" with a stair of blocks of increasing lengths. The combinations making 10 and the corresponding subtraction facts are grouped together in the Analysis of 10 (which shows the component parts of 10). From the study of the Pattern Boards, the picture of the even numbers as "doubles," $1+1=2$, $2+2=4$,

etc., is idelibly impressed on the child's mind, as are the corresponding subtraction facts: $2-1$, $4-2$, $6-3$, $10-5$. With the same clarity, the child visualizes the series: $1+2$, $2+3$, $3+4$, $4+5$, that constitute the odd numbers, 3, 5, 7, and 9. What he thus puts together in his mind, he may take apart with equal facility: $3-2$, $5-3$, $7-4$, $9-5$.

If a child now faces mastery tests at the end of the workbook for grade 1, he will write down the correct answers instantly; but, by no means, are these automatic responses due to blind memorization of single facts. If the child should waver at any point, he will always be ready to visualize to what group the number fact belongs and thus he can reconstruct the answer in his mind. A few "structural techniques" ensure mastery and make the child confident and secure at the end of the first grade. A guide to "problem solving" takes the "problem" out of this art, too, so that the child is able to solve problems in addition or subtraction within the limits of 10.

Numbers Beyond 10

After the child is at home in the range of 1 to 10, we lead him to the new and fascinating step of reaching the numbers beyond 10. We have only the numbers from 1 to 9 and the zero to denote all the quantities which we want to conjure up in our mind. How is it done? The Roman numbers (XV, e.g.) show that a fifteen was built up by 1 ten and five units. We do want the child to discover the exciting step to two-place numbers himself. But in the experiments that will follow, the child will gradually advance to the full understanding of the ingenuity and simplicity of our number system which an unknown Hindu gave to mankind when he invented "positional notation," or the "place value" of a number.

In the first experiment of this series, we give the child a flat square empty box (the 20-Case) and put two sets of Unit Blocks before him (i.e., two sets of blocks from 1 to 10) and a set of ten 10-Blocks. We ask the child to build a stair of blocks. When he reaches 10, he stops, usually saying: "Where do you keep the longer blocks?" We do not have any blocks longer than the 10-Blocks, so we ask him to proceed with the stair, using the blocks at his disposal. Tentatively, the child puts in another 10-Block and adds a 1-Block on top. "It works," he says, "I just piece them together." He builds the stair with $10+1$, $10+2$, $10+3$, and so on. "Look," he says, "the same stair—one flight up!" (See Figure 5.)

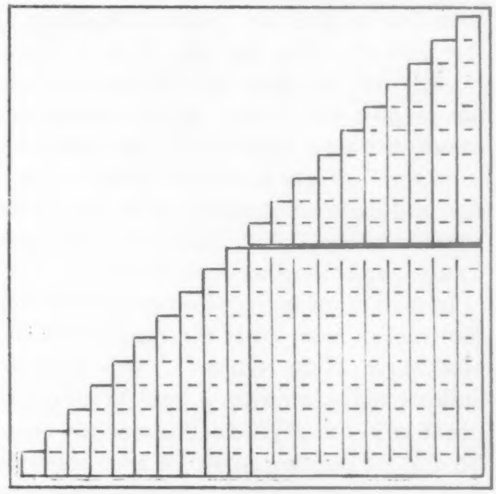

FIG. 5. The 20-Stair in the 20-Case.

Only after the child has constructed the steps of this stair by himself do we tell him the names of the new numbers. A 4 and a 10 are called 14, a 7 and a 10 are called 17. That makes sense, and 16 and 19 are well-named, too. However, 15 and 13 are often called "Five-teen" and "three-teen" by the child. To the child's logical mind, eleven and twelve are misnamed; they should be called "one-teen" and "two-teen." After the structure of the "teen-numbers" is fully understood, the child may start experiments of addition and subtraction in the range to 20, and, as he tries the "Climbing 1" or the "Climbing 2" up to 20, he will discover the "+1" and the "+2" facts in this range.

Although the "piecing together" of tens and units is one of the fundamental prin-

ciples of the number system, we need another demonstration to introduce "positional notation." The experiment which develops the notion that 3 tens are 3 "pieces" just as 3 cubes are 3 "pieces," but different in size, is called the "Store Game." Give one of the children a "store" of 9 units or cubes, another child 9 ten-Blocks. The teacher puts on the desk a "holder" for number cards for two-place numbers and explains the game: If she puts a 3 to the right in "units' place," the child who has the units has to give her 3 cubes. However, if she puts the 3 to the left, it is in "tens' place," and the child who handles the tens has to give her 3 tens. As the game continues, the class understands the difference in "value" or size which the place or position of a numeral indicates: each time there were 3 "pieces," but once they were just 3 cubes, and the other time, 3 tens.

Instead of using the number card holder, the teacher now writes the number on the blackboard. If she writes "3," how does she indicate that she means 3 tens? What could put that 3 in tens' place? The teacher writes 30, and the pupils see that the zero indicates that 3 *tens* are wanted. The zero plays the role of a "place holder," but at the same time, it means "*nothing*" in units' place, or, no units. So the number 30 calls for just 3 tens. If we now get 35, both children get to work: we need 3 tens and 5 units. Only now are the names of the new numbers introduced: 2 tens and a 5 are called thirty-five, 6 tens and a 3 are called sixty-three, and so on. The largest two-place number is 99, i.e. 9 tens and 9 units. One more unit gives another ten, so that we have 10 tens in all, which we call 1 hundred and to which we assign a new place. At this level, we shall study the range from 1 to 100. The next step is then to acquaint the child with the location of each of these numbers and the amount that they represent.

We place a Number Track from 1 to 100 in front of the pupils, in which each place may be marked by the wire bridge (called: Jumper), and in which the blocks may be inserted. Let us place the Jumper on 35. How can we reach 35? Either, we fill the track with 35 units (or cubes), thus learning what amount 35 "calls for" be it peanuts, or pebbles, or pages in a book. But it is a tedious job to insert that many cubes. We had better select 3 tens and a 5-Block, and here we are!

Experiment after experiment makes the child familiar with the new number range. If he holds the Jumper in his hand, and the teacher calls: 19, 61, 89, etc, the Jumper comes down at the right place in a second: the pupils know where each of these numbers "live."

Extending Number Combinations

The study of addition in the new range is full of exciting discoveries. Put a 10-Block into the Number Track at any place and see what "adding 10" means. If you add 10 to any number you reach the same place in the next higher decade: $23+10$ equals 33, $86+10=96$, and so on. Here is another interesting fact: everything we learned in the handling of units, holds true for the tens. If 3 plus 2 are 5, this must be correct for apples, giraffes, or tens, also. Thus the pupils find: $30+20=50$, when they add 2 tens to 30 in the Number Track. There is still another experiment; look at the first decade, the range from 1 to 10. Whatever you know about this range, holds true in any of the higher decades. If $5+3=8$, you may travel with the two blocks through the whole track and "8" will be reached each time: $65+3=68$, $95+3=98$, and so on.

Yet there are some new number facts which were not encountered before. When the children tried to fit the 4-Block on top of the 7-Block in the Unit Box, it did not fit in: 7 plus 4 were bigger than 10, so the 4-Block had to be discarded, and there was room only for the 3. Now, we shall take up just those facts in which we go beyond the 10-limit, i.e., when we bridge across 10. A few structural techniques in addition as

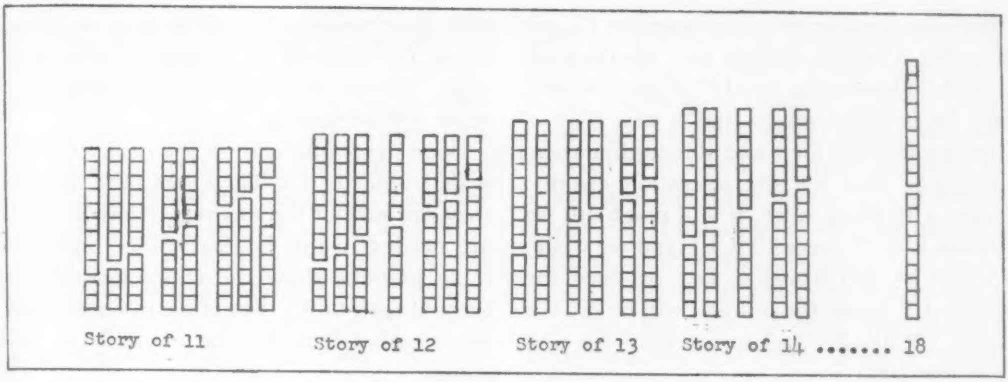

FIG. 6. The bridging facts.

Story of 11:	2+9=11 9+2=11	3+8=11 8+3=11	4+7=11 7+4=11	5+6=11 6+5=11
Story of 12:	3+9=12 9+3=12	4+8=12 8+4=12	5+7=12 7+5=12	6+6=12
Story of 13:	4+9=13 9+4=13	5+8=13 8+5=13		6+7=13 7+6=13
Story of 14:	5+9=14 9+5=14	6+8=14 8+6=14		7+7=14
	. . .	. . .		. . .
Story of 18:				9+9=18

FIG. 6. Numbers Beyond 10

well as in subtraction teach the child these "bridging facts." There is one impressive experiment with the blocks that shows how we may go from the Analysis of 10 (The Story of 10) to the Analysis of 11, 12, 13, 14, 15, 16, 17, and 18. In the experiment, we move the upper set of blocks "one up" each time. The result is briefly shown in the next figure. (See Figure 6.) Each analysis shows the component parts of the number that is studied. In this "decomposition," the pupil meets striking facts which he will not forget. If he knows that e.g. 5 plus 6 are 11, he may think the parts together: $5+6=11$, or he may think them apart: $11-6=5$ and $11-5=6$.

At the end of grade 2, there should not be a child in the class who is not sure of the 100 addition facts and the 100 subtraction facts. The application of these facts in "problem solving" does not cause any difficulties either. We are ready for the next important step.

A pupil who is sure of the bridging facts, discovers with interest how this knowledge is used in "carrying" and "borrowing." The experiments are carried out with the Dual Board which has a Unit-Compartment and a Ten-Compartment. If the total that results from adding units exceeds ten, a ten-Block is actually *carried* to the Ten-Compartment where 1 ten must be added to the total number of tens. Similarly, the operation of actually carrying back a ten-Block to the Unit Compartment if it is necessary to "borrow" 1 ten parallels exactly the algorithm which we use in the written work.

To study multiplication and division, we return to the Number Track from 1 to 100.

Since our number system is based on 10, our Number Track is divided into ten decades. The child knows the "peaks" of the 10-Scale, and he is delighted to find a new way of wording familiar facts and of recording them in a way that fits the operation exactly. Insert a 10-Block once, it will reach 10; we express it as: 1 times 10 is 10, and we write: $1\times10=10$. Do it twice, i.e., produce the 10-Block 2 times: the product will be 20. We write: $2\times10=20$, and so on, until we find $10\times10=100$.

The 5-Scale is almost as simple to grasp. Since two 5's are 10, the pupil discovers with pleasure that 2 five-Blocks reach 10, that 4 fives reach 20, etc. The "odd" facts, i.e., 1×5, 3×5, 5×5, and so on, lead to the middle of the decades, so: $1\times5=5$, $3\times5=15$, $5\times5=25$, and so forth. The facts of the 5-Table soon become unforgettable. The products of the 9-Table follow a "trick" of their own which we demonstrate with the Dual Board. We show that 3 nines are 3 tens less 3, or, $30-3$, or 27. Six nines are seen clearly as 6 tens less 6, or 54. At this point, the children say: "We've got the hang of it." The 2-Table means *doubling*, its results are the even numbers. Every second peak in the 2-Scale is a peak in the 4-Scale, every second peak in the 4-Scale is a peak of the 8-Scale. (See Figure 7.) A look at the picture will show how easy it will be later on to develop the concept of *common multiples* and, thus, *common denominators* when these concepts will be needed.

Soon the children know the peaks of every scale from 1 to 10. They love tests because they are sure of the answers. No drill cards are needed in this method. Since unforgettable pictures show the characteristic features of each table, the child can reconstruct any number fact that he might have forgotten.

The undoing of multiplication is division in which we ask how often one number is contained in another one. I showed one boy that $4\overline{)32}$ is to be read as: How many 4's are there in 32? He not only answered "8" but proved this fact by inserting 8 four-Blocks in the Number Track. After he solved all the examples of dividing by 4, he said as an explanation of his quick work: "My 4-Table helped me."

The Number Track makes it easy, too, to grasp uneven division. The Jumper is placed on any number, e.g., on 45. How many 8's are in 45? The pupil thinks: what is the nearest 8-peak to which I come before going up to 45? To reach 45, he will place 5 of the 8-Blocks into the track (reaching 40), and he fills the remaining gap with 5 cubes. So, 8 goes into 45 five times and a remainder of 5.

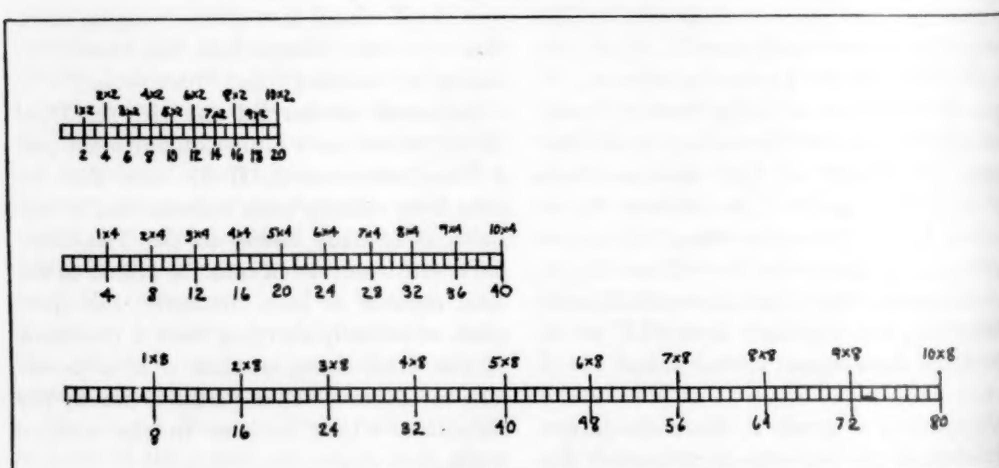

FIG. 7. The 2-Scale, 4-Scale, and 8-Scale.

The first steps of *long division* are understood: we always look for the nearest peak, or, the nearest "multiple"; we write the quotient i.e. the number that tells how many times the divisor is contained in the dividend, and then we figure out the remainder. Thus, the Number Track teaches how we find products or multiples, and how, in reversing the procedure, we find quotients.

There is another important counterpart to multiplication which is partition. We introduce partition with a new set of materials, using a different terminology and a new way of recording what we find. In these experiments, the pupils find the size of the part or share if, for example, 12 is divided by 4. He finds that the answer is 3, and he records: $12 \div 4 = 3$. After some such work is done, the teacher shows that we may move *one* of the parts away: 1 of the 4 equal parts of 12 is 3. We write it: $\frac{1}{4}$ of $12 = 3$. If we move two such parts over, we'll find: $\frac{2}{4}$ of $12 = 6$. The pupil takes over and discovers: $\frac{3}{4}$ of $12 = 9$, and $\frac{4}{4}$ of $12 = 12$. With the last move, he has put Humpty Dumpty together again: he has re-assembled the 4 fourths to make the whole.

In the course of this study, the pupils delight to do some detective work which incidentally is the first step towards the understanding of "Case III" in percentage. The teacher fills one of the partition-dishes with 4 cubes and says: "I had a number of cubes and divided them by 6. Look here: $\frac{1}{6}$ of the amount is 4. Who can tell how many cubes I had at the start?" The detective has a clue: there were 6 parts, so he selects 6 dishes. In one of them he puts 4 cubes, so into each of the others go 4 cubes also. There must have been 24 cubes at the start! We may even write it down: we may call the unknown amount at the start N. We know that $\frac{1}{6}$ of $N = 4$. It makes sense that the whole, or 6 sixths, must be 6×4, or 24.

At the end of grade 3, in our course of teaching, the children know carrying and borrowing, multiplication, division, and partition, i.e. finding fractional parts. Once the operations are mastered, the pupils attack problem solving with equal understanding, a theme too broad to be handled in this paper.

Structure of the Number System

In the next step we are ready to study the structure of our number system based on the powers of 10, moving from units to tens, hundreds and thousands. We offer the pupils materials with which they can construct *any number* between 1 and 1,999. According to our number system, we need 1 Thousand-Cube, 9 Hundred-Squares, 9 Ten-Blocks and 9 cubes or units. With these few pieces, any number may be constructed—as we only have the figures from 1 to 9 to write them. There are two tasks to make the pupil conversant with the numbers of this range. You may place 1 Thousand-Cube, 5 Hundred-Squares, 9 tens, and 3 cubes on the table, and the pupil is to name the number and to write it: 1,593. Or, you may write a number

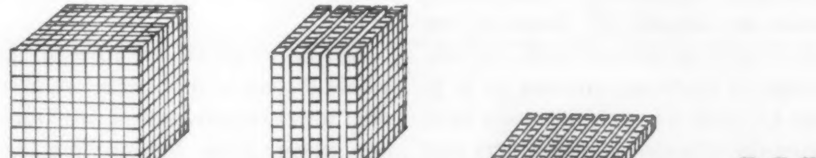

FIG. 8. The number 1,593

like 1,215 on the blackboard and have the pupil build it with the materials and then name it. As an example, see below how the number 1,593 is constructed with the materials. (See Figure 8.)

Of course, the children find it more interesting if there are some "empty places," if, for instance, the Thousand-Cube is placed before them with just 1 little cube—no hundreds, no tens—representing the number 1,001. Or, if the teacher writes 1,040 on the blackboard, the pupils build it with 1 Thousand-Cube and 4 tens, naming it 1 thousand and forty.

Carrying and borrowing in this new range is most exciting. Let us build the number 999 with 9 hundreds, 9 tens, and 9 cubes; if we now add 1 cube, there are 10 in units' place, so we carry 1 ten; that gives ten 10's, so we have to carry 1 hundred; this gives 10 hundreds, so we have to exchange them for 1 Thousand-Cube.[4] The corresponding example, to take 1 from 1,000 is as thrilling an experiment. Compound Multiplication and Long Division can also be made clear with the concrete materials, but these experiments would lead us beyond the scope of this paper. We show the child with special emphasis that multiplying by 10 changes units to tens, tens to hundreds, hundreds to thousands, and so on. If we write the numbers, we see that a number multiplied by 10 moves "one place up." This procedure will be of decisive help in the understanding of decimal fractions.

We can think of the *1* in our number system as a *swinging point:* if we take the number 1 three times or five times, or a hundred times or a thousand times, we arrive at 3, 5, 100, or 1,000 respectively. These are the *whole numbers* or integers. If, however, we divide *1* by 3, or 5, or 100, or 1,000, we create the realm of fractions, arriving at $\frac{1}{3}$, $\frac{1}{5}$, 1/100, and 1/1,000. There is not space here to deal with these fascinating numbers and show the materials with which the pupils discover the laws that guide their operations.

[4] "Concrete Presentation of Geometric Progression," *The Mathematics Teacher*, Vol. XLIV, No. 3, March 1951, by the author.

But we must, at least, mention how we explain "decimals" and "per cent."

Decimals and Per Cents

In the usual teaching, decimals are introduced as a special subject. In the eyes of the pupils, decimals are exasperating in more ways than one since they have to worry about the position of a very "unstable" decimal point! This decimal point moves now to the right and now to the left—one move is connected with multiplying the number by 10, and the other with dividing by 10, and its position does not seem to make much sense. This treatment of decimals is a violation of the unity and continuity of our number system. In teaching STRUCTURAL ARITHMETIC, we show the pupil the tiny pieces that result if we cut the small 1-Cube into 10 slices, each of which is called 1/10. If these slices are cut in 10 rows, each row is 1/100 of the cube; and if we now cut this row into 10 tiny cubes, we have miniature cubes that ate 1/1,000 of the 1-Cube. A number like 1.593 designates 1 unit, 5 tenths, 9 hundredths, and 3 thousandths, or, 1 and 5 hundred ninety-three thousandths. If we would look at it through a magnifying glass, we would see the picture presented in Figure 7. Thus the pupils may construct decimals with our materials. In experiments with these devices he discovers that what he learned with the big numbers must hold true for the miniature edition also. If we multiply a decimal fraction by 10, it means, of course, that we change each denomination to the next higher one. If we, for instance, multiply 1.5 by 10, units grow into tens, tenths grow into units, i.e. the number "moves up" one place; the result is 15.

Should we divide by 10—throughout the whole system of numbers—we arrive at the next lower denomination, we "move down" one place: from the millions to the 100 thousands, from the tens to the units, from the hundredths to the thousandths. It makes the pupil confident and secure in his work if he sees that we have one homogeneous number system with understandable rules and

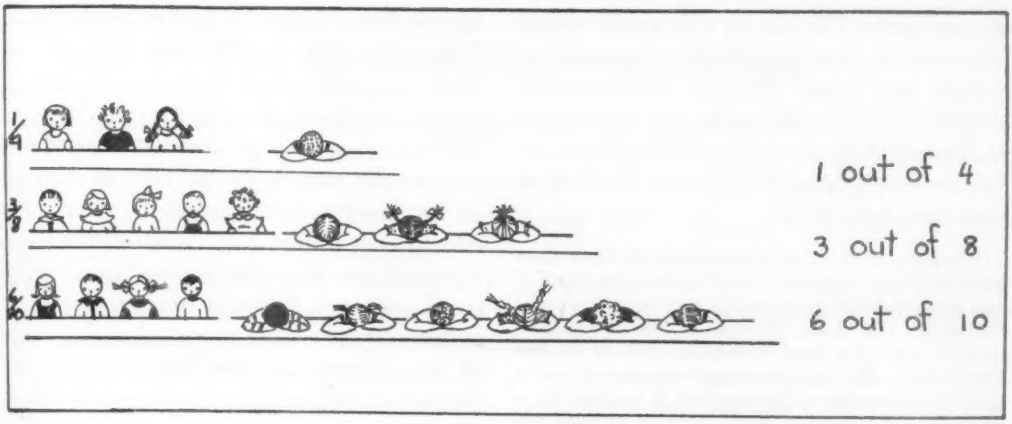

FIG. 9. Fractional parts.

laws that are consistent throughout—from the millions to the millionths.

Hundredths are part of this uniform system, too, and they should not be handled as another strange and separate item, called "per cent" and belonging somehow to business arithmetic. After we have studied per cents in their role of hundredths, we use still another approach to reach complete understanding of the meaning of per cent. Suppose 1 out of every 4 students goes to sleep in a lecture, that would be $\frac{1}{4}$ of the audience. If, in the next room, 3 out of 8 doze off, that would be $\frac{3}{8}$, and if in still another classroom, 6 out of 10 close their eyes, that would be 6/10. (See Figure 9.)

If the teachers want to compare the state of sleepiness among the students, they have to compare the fractions $\frac{1}{4}$ and $\frac{3}{8}$ and 6/10. It is not difficult to find a common denominator and to change the fractions accordingly; but we can spare the effort. All we have to do is to go back to our Number Track from 1 to 100. Let us measure: $\frac{1}{2}$ of the track from 1 to 100 reaches 50, $\frac{1}{4}$ reaches 25, $\frac{1}{8}$ of the track is $12\frac{1}{2}$, and so on. (See Figure 10.)

If, in the above case we have to deal with $\frac{1}{4}$, that means exactly 25 out of 100, $\frac{3}{8}$ is represented by $37\frac{1}{2}$ out of 100, and 6/10 indicate 60 out of 100. Now, 25 out of 100 is called "25 per centum" or 25 per cent, or 25%. We get a clear picture in our mind what portion or fraction or percentage of the audience went to sleep in the different classrooms: 25%, $37\frac{1}{2}$%, and 60% respectively.

Unfortunately, there is no room here to go deeper into the questions connected with ratio or proportion although more sins are committed in their names in the usual teaching of arithmetic than in any other sphere of mathematics. We can't show how our Number Track may be extended "down from zero" to give pupils the opportunity of discovering the realm of *negative numbers*. Nor can we indulge in experiments that would show how our blocks may be used to solve simple equations, or to find squares and

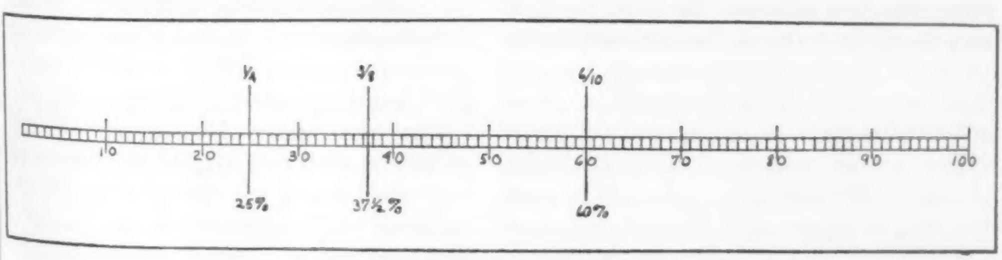

FIG. 10. Changing fractions to per cent.

square roots. The aim of this paper was to demonstrate that structurally adequate materials can make abstract numbers and number relationships *visible* and so concrete that every child can grasp the simplicity and beauty that arithmetic holds for all of those who have eyes to see.

EDITOR'S NOTE. The Stern materials have been used by many teachers to lead children to discover and learn number relationships and "facts" or combinations. The basic concept of her blocks and sticks is more of a linear dimension than of the collection idea. She has questioned whether or not a child in arranging 6 buttons and 2 buttons for a total of 8 is restricted to a visualization but one disconnected number fact. This need not be the case. In fact with all materials whether they be textbooks, workbooks, or manipulative devices it is the method of use that is of prime importance. Miss Stern organizes her use of materials carefully so that children discover and think and see a visualization of concepts and relationships before they enter the mastery stage. In a sense this is a mathematical approach in that the pupil goes from numbers to the use of them with buttons, chairs, pupils, and other problem applications. The materials when used by a competent teacher ought certainly to be very helpful in the formation of good mathematical ideas and to good thinking with numbers and the relationships of numbers and processes. Certain teachers need to be counseled to restrict the use of manipulative devices when this is no longer necessary or desirable.

The Stern materials and books based upon their use are available from Houghton Mifflin Company.

Filmstrips Reviewed

Arithmetic Problems, Creative Education, Inc., 340 N. Milwaukee Avenue, Libertyville, Illinois.

(1) Problems in Addition
(2) Problems in Subtraction
(3) Problems in Multiplication
(4) Problems in Division

These filmstrips are designed for study of the basic facts. Each is divided into two parts, the first contains the easy facts, an easy fact in each frame, the last contains the harder facts, a harder fact in each frame. These filmstrips are excellent not only for study purposes where they may be used by children individually, but also they may be used as means of evaluation where children in the class may write the correct response on his paper as he sees it on the screen. For effective use in this way, the filmstrips are so designed that they may be used in a lighted room and each frame is numbered so that children will be able to match the number of the question on the filmstrip and the number of the answer on their paper. These filmstrips are another means of providing experience whereby children may be able to master facts associated with the basic operations.

E. GLENADINE GIBBS

New Arithmetic Textbooks

The following series of textbooks have been received.

1) Iroquois Publishing Company, Syracuse, New York: Numbers at Work Series, Grades 1–8, David H. Patton and William E. Young.
2) Laidlaw Brothers, River Forest, Illinois: Understanding Arithmetic Series, Grades 1–8, E. T. McSwain, Louis E. Ulrich, Ralph J. Cooke (textbooks, workbooks, teacher's editions).
3) World Book Company, Yonkers, New York: Growth in Arithmetic Series, Grades 1 and 2, John R. Clark, Charlotte W. Junge, Caroline Hatton Clark (teacher's edition, pupil's workbooks).
4) Allyn and Bacon, Inc., Boston, Massachusetts: Arithmetic in My World Series, Grades 1–6, C. Newton Stokes (teacher editions, workbooks, student books).

The Littlest Mathematician

An Approach Applicable at all Levels

MARGERY BAUMGARTNER
San Diego State College, Calif.

DOES NUMBER COVER THE WHOLE AREA of mathematics? Does spelling cover the whole area of language arts? Numerical computation is only one part of the study of mathematics. From a functional viewpoint, it is important to broaden the teaching field to include those aspects of mathematics a student will use after graduation—when machines have taken over much of his computation.

To the teacher who loves to teach, it is a matter of chagrin to recognize that the graduate needs only sixth grade reading ability to read his newspapers and magazines, and that he seldom does any complicated computation. Nevertheless our average citizen needs to read intelligently enough to draw the right conclusions; and, computing aside, he acts with regard for mathematical ideas over and over in his daily life.

The teacher who hopes to produce real growth in all aspects of quantitative thinking, to develop discriminative behavior with quantity, to awaken appreciation of interrelationships, *must learn to listen and observe as well as tell and test.* Essential it certainly is to look conscientiously at one's own functioning. But the arithmetic teacher would not exist without the arithmetic learner. Let's study him for awhile.

When I entered first grade, I am sure my teacher regarded me as a blank page on which she would write everything I needed to know. (After a time unfortunately something got smudged!) Now we know that even the six-year-old comes from somewhere, bringing his past into the schoolroom with him. And the older he grows, the longer his history. So at the upper grade levels as well as the lower, there is no text that can take the place of the teacher's really knowing and continually analyzing what is going on in the learner's mind.

In the upper grades the child is judged largely on number work, and even that mostly in the written form. There is a need for analyzing the child's behavior with mathematics—what he does, as well as his numerical computation and linguistic skills. His behavior should be judged in terms of the depths of meaning he has developed.

The great difficulty in the teaching of arithmetic is that the pupil learns the words, the processes, faster than he masters the principles underlying them. Verbalism has set in. I felt that all my teachers wanted from me was the right answer, so I faithfully "carried" and "brought down" as instructed. The idea that I should know what I was doing apparently entered neither my calculations nor the teachers'.

When a child has been given too little experience and too much verbiage in the beginning, the first teacher to recognize the deficiency has the obligation to provide the very elements which were missed at the lower levels: a rich environment, many real experiences, opportunities to express what he has found out for himself without meaningless memorization. *The cure for malnutrition is enrichment.*

Of course, an ounce of prevention is better economy than a pound of cure. How can we tell whether a child understands what he is doing? *Strategic watching and listening by the teacher can help prevent verbalism.* How far back would we have to go to observe behavior unprejudiced by language? To the beginnings of language, or farther? There is evidence that reaction with regard to quantity begins with conscious life. I am actually pro-

posing to look all the way back, to a student none of us will be called upon to teach: the infant, who is teaching himself, by a method I respectfully call "pure research."

What possible practical application to the teaching of sixth grade arithmetic can we hope to find in the observation of a baby, too young for any school? I believe there is a learning experience for us here. Watching an untaught child finding out for himself has more than theoretical interest. It suggests a *method* that leads somewhere.

Characteristics of Learning

Specifically, the Littlest Mathematician may remind the teacher of older children of these principles which slip from our consciousness too easily and too often:

First, the child is mathematicking in the broad sense not only when he is counting but whenever he is exploring any relationships of quantity—numerical quantities or spatial quantities.

Second, maturation is promoted if someone is there to lend an ear and perhaps occasionally a hand—to ask questions which will deepen the child's interest in his own explorations. Throughout the elementary school, listening to students thinking aloud is a valuable diagnostic device. Learning to look at the child's response to experience will guide us in knowing what questions to ask to lead him to higher levels of discriminative thinking.

Third, the child is absolutely dependent on the environment (and *this means you*) to supply the raw materials for the experiences on which concepts are based.

Seeking the origins of discriminative behavior, we can trace continuous development of concepts of differentiation from the "booming buzzing" stage to various areas of mathematical discrimination: motion, change, causation, form, dimension, time, order, variation, relation—and number.

Number comes last because the mathematics of the nursery is something larger than arithmetic—it is the scientific approach to life. The infant is obliged to be a research scientist if he is to learn in his first few years all he will have to find out that no one can teach him. It is a challenge to his later teachers to keep alive his own infinite interest in exploring every aspect of life that comes his way.

"Watch, for example, the baby who has suddenly seen a new object in his path, say, a small stick. Instantly the stick becomes the object of his whole being. His attention is focussed and riveted for the moment while he slowly advances on it. The whole process of scientific research is brought into play in order to translate and integrate this new discovery. No laboratory technician could conduct the experiment with greater finesse and with more elaborate attention to detail. First, the eyes watch, the hands slowly reach out, the fingers touch lightly and feel; then the hand grasps. The stick is turned over and over for visual examination, and it is given the taste test. It is pushed, pulled, beaten, shaken, hammered, bounced, rolled, tossed and dropped. The process may be repeated depending upon the intensity of his absorption and his ultimate satisfaction with the experiment. Each new discovery becomes a positive experience enriching the meaning of life to the child. Watching, we become aware of the value of his experience and how important it is that we encourage it."[1]

When the child comes to attach a name to this object, he will know what he is talking about because the symbol will rest firmly on rich experience with the reality. He is ready to learn the word "stick"; but "brotherhood," for instance, will be only a word if he learns to say it at all. Understanding will wait upon experience.

We may get the child to repeat our mathematical formula $(2+2=4)$ but we ought to know whether he knows what he is talking about, whether he believes what he is saying. Success in the teaching of mathematics will come faster when names are attached only after non-verbal awareness of the concept is established.

[1] *The Challenge of Children*, by Cooperative Parents' Group, Whiteside Morrow: New York City, 1957.

The infant is constantly at work developing the pre-verbal understandings that will constitute his readiness for academic learning. Without reading it in this morning's paper, he knows he needs to study science, math, and foreign language, and that is exactly what he is doing (without neglecting human relations, either).

Case Study

This record is based on continuous informal, unplanned observation of a single child in the home studying his science, math, and the foreign language which our native tongue is to us all in the beginning. Would anyone choose to go to college if the curriculum for four years were anything like the load every child must take on in his first four years?

This account will describe only the expressions and thought patterns for the second and third years. Note the groping with mathematical ideas and how concepts are being formed.

Number: The Littlest Mathematician had too many other things to learn to spend much time on number, even though he picked up the number names easily while learning to talk. At 2 years 1 month (2;1)* he enjoyed repeating the numbers to ten after an adult, very animated and sometimes anticipating the adult. While scribbling he sang to himself "6 7 9 10 11." The first complex idea and compound sentence came when his mother gave him the next to last slice of watermelon on the platter and he remarked "There's another—and big!" (Hunger provides fine motivation!) She gave him raisin bread, which he calls cherry pie, and he picks out all the raisins to eat first, saying, "'Nodder one. More one. Cherry pie all gone."

At 2;3 he can accurately beat the drum rhythms for his record Little Indian Drum: "Where is Red Fox? I am here!" although his verbal rendition is "Where is Red Box? I am here-y."

* 2;1 indicates 2 years 1 month. This form will be used throughout the manuscript.

3;2 His stories contain numbers for their own sake: "D'you never'd see a boy with ten feet and leventeen fannies?" "Once I gived a dog two bones—I mean flee."

3;4 When tactfully asked if he preferred to go to bed piggyback or frontwards he had the presence of mind to reply "No wards."

3;5 "My family's going each away and I want somebody to love me!"

3;5 "The bullsdover did a lot more work —just the same work and a lot more."

3;5 Drawing with a handful of crayons all at once: "Shall I use all the colors instead of two?"

3;11 Hearing of a cat's nine lives: "Nine lives! That's a lot of kittens!"

Time: At 2;1 he often used the phrase "last night."

2;3 He has a record story in which people have to be called for school and work when Frere Jacques fails to sound the matins. Half an hour before dawn he came to his parents' bed and lay quietly until daybreak, then said, "Wake up, Daddy. It's working time."

2;5 "Did the kitty come in yet?"

2;9 Something reminded him of a visit to relatives last year. "We were in Skipper's house a long time of go. And we started to go, and you started the motor, and we did."

3;2 "I don't like to go very often. I just like to go easily, a little bit."

Size, weight: 2;1 Pushing Cheerios into mother's mouth like pennies into a bank, he differentiates the overdone ones which are a trifle smaller, saying, "Eat the 0-0's. Baby 0-0. Big one."

2;2 "I carry dese records. I carry heavy-big records."

2;3 He asks himself "How does the dog go? Wow, wow. How does the *little* dog go? Woo, woo." He makes a tiny sound for the little dog. This seems to be his own invention, not a learned game.

2;9 "I saw the birdies in their little barns. I saw a long blue bird with bright, bright feathers on him."

3;5 "I'm drawing an auchtrich. Here's his big beak. See that new wide feet of his own? He has to wear a lot of back feet, because he has to walk and walk around."

3;8 A very complex story, with language under control: "Do you know, when someone was passing (juice), Angie got a little glass and I bot a big glass; and Beth, who was sitting next to Angie on the other side, got a glass almost as bit as mine!"

3;11 "My tonsils will be as big as walrus tusks!"

Growth: 2;6 Finding his outgrown shoes, he realizes how much he has grown. "These are Carl's baby white shoes. Isn't that cute, Mom?"

2;7 He resists taking his Vitamin D and mother says "You can count your drops—you're a big boy." "No! I'm just a baby."

2;9 Father says, "You used to hang these beads on my ear when you were a baby." "But I'm a big boy now— they go on your wrist."

2;9 His cousin sent him an outgrown sweater. "Skipper tried and tried and the sweater was too little, and I tried and it was just right."

2;10 Visiting a farm, he said "I'm too thin to pat that horse."

3;2 "Good night, children—the big children and the little children—the grownups and the little people—grownups and the little growndowns."

3;4 "I'm growing down. I was too big. I'm just a teeny baby, with no teeth."

3;4 "When will I be a boy?"

3;6 "I don't want to grow up. I want to stay little. I'll be 4 years old, then 5, and I'm getting too big for all you people."

3;7 "Am I going to be as big as Grandfather?"

Grouping, similarity, belonging, contrast: At 2;1 he discriminates between cookie and cracker—and after finishing a cookie given by a neighbor starts back toward her house saying, "I go see a lady."

2;1 In a heterogeneous group of pictures he points out those things which have motors.

2;3 Likes to assemble things: "Whess a wheel go on? Which goes dese goes on?" His language is so well under control that he likes to call things by the wrong name as a joke—calls the doorstop "chicken meat."

2;8 Brings mother her glasses: "You put these on; they won't see for me."

2;8 Reading picture book: "One mommy possum's walking without some shoes—she's walking with some feets." He is greatly interested in contrasts and humorous appositions. Says, "You read to me—you're Dr. Reed!" (his pediatrician). After a fruitless car trip: "We want to find Daddy in his *work*, and he was in our *house!*" He contrasts happenings at home and at nursery school: "Mary Jo pushed me into the table yestiday. *You* pushed me in my *house*, and you pushed the *table* in to the *chair!*"

Position, form, completeness: 2;1 He puts a small airplane on a larger one, saying "Airplane piggyback."

2;2 He has so far operated the phonograph by turning it on, set for automatic play, and just toying with the manual controls. Now he has learned to turn it on, place tone arm, turn off, lift and remove arm to side. He has also begun to arrange things artistically, spreading a tissue on the kitchen stool saying, "Sit down, Mommy. That's nice, Mommy. This is a nice kitchen, Mommy." He arranges tiny pieces of tissue on mother's arm, then soberly spreads a large piece over all. Having no building blocks at home, he makes a very neat row of sandwich bags across the floor, stepping over it carefully. In his dollhouse he spreads washcloths as rugs and arranges tiny cars or dolls on them, saying, "I like a dollhouse." He himself sits with his toy kangaroo in an enclosure made of a folding cardboard panorama book, which he has used in this way since babyhood.

2;9 Seeing two dogs on a hillside: "One is sitting high and one is sitting low."

2;9 At dinner he talks to himself: "See my meat, Carl, before 'That's all'? See it Carl? That's all." Eats.

2;9 He likes things done in proper form, frequently saying "That's right, Mommy." He tells a story with beginning, middle, and end: "We went to the station and it was a train! A dingdong was saying a good noise. No whistle on the black train this time. My puzzle train has a red whistle. Now it was going ot Philadelphew. And so I went back

to Play School. Something was coming next. And it was tomato juice!"

2;11 He reads himself a picture book as he thinks it should be read, clicking his tongue importantly at the beginning of each sentence: "*One* day—there was a cow. And *one* day—there was a calf. And *one* day—"

2;11 Dancing to his favorite Prokofieff concerto, he listens for the parts he particularly enjoys. Takes his parents' hands and goes around in a circle, then separates, saying, "Now let's dance without." Stops just to listen to a passage, then says, "Now let's dance this faster one. Here comes the toten-ton-ton-ton." Flexes knees, ready to jump when the music changes; leaps, then steps strongly from one side to the other on the repeated sforzandos.

3;4 When tired of seeing flower show, says "I'm all finished." Dining alone with friends, he resists the suggestion he phone for transportation home until suddenly he speaks up in a small voice, "I'm finished."

3;5 Carl has made up a riddle: "What's this big and can't walk?" What is? "A boy." Why can't he walk? "Just one leg."

3;6 Speaks of "the whole night long." Tells about an evergreen forest: "A tremendous pile of Christmas trees! One touched the sky. It was bigger. I saw it at the very last end."

3;6 "How we cooked the punch (i.e. fudge): We put butter on the two cooker things, round—one pan under, the other on top. Instead of being all rumbled up, it smoothed all out, melted all around."

3;11 "My parents are you and Daddy and——my double good, my very best parent is Grandfather."

Conclusions

We have taken a quick look at the quality of one person's quantitative behavior. Our little student has finished four years of undergraduate study, beginning with learning to breathe for himself, sit up, catch on to the language and the customs of the culture. We may graduate him cum laude on the basis of the culminating works of his junior year and senior year. (2;8 "Mary Jo pushed me into the table yestiday. *You* pushed me in my *house*, and you pushed the *table* in to the *chair*." 3;8 "Angie got a little glass and I got a big glass; and Beth, who was sitting next to Angie on the other side, got a glass almost as big as mine.")

We might have guessed he was a better than average student when, confronted with two choices, he thought up a third alternative: neither. "Piggyback or frontwards?" "No wards." Children do this so rarely that I have come to believe it must be related to the slowness of the human race in inventing the concept of zero.

Our graduate will not present a pristine page for the kindergarten and first grade teacher to write on when he begins this post-graduate course. What he needs is guidance for what is already at work in him. He shows readiness, and the student who is ready should be encouraged to go ahead. What will happen now? (Actually neither teacher nor parent did offer this child the opportunity to move ahead at his own rate in mathematical development; and that is why we have been moved to write the story.)

What do this little student's efforts at problem solving suggest to the arithmetic teacher? First we recognize that the quantitative behavior, the mathematics actually used by the young child, does not consist mainly of number, sometimes the only aspect the primary teacher thinks of presenting.

Can we then watch what the child himself is working on spontaneously, so that the curriculum includes what he wishes to learn as well as what we wish to teach? In order to know what teaching is needed, we must listen, and sometimes listen even with the third ear, and after teaching, listen again to see how effective the teaching has been. Before we chug on ahead, let's find out if the learner is on the right track, and if so, how far along he is. His achievement may be very slight, but it should be solid. A few little numbers used with confidence are better than a course in trigonometry lost in oblivion.

The object of teaching is not to go through

processes but to understand principles. It is not only permissible but preferable to have non-verbal awareness before pinning down processes with labels. It is the teacher's job to supply the experience necessary to invest symbols with real meaning for the students.

The Littlest Mathematician has no prejudice against arithmetic—yet. He grapples with mathematical problems for the sheer joy of it. It is fun, just as stretching his muscles dancing to the music is fun. There would be no shortage of people wanting to go into science and mathematical fields if the natural delight of the child in these phenomena could be maintained. Some effort is made in the schools to convince children that music and Shakespeare can be fun. Mathematics is rarely taught in this way. The teacher's cue should be to keep alive the spirit of exploration—the scientific approach—and to preserve the concentrated interest and delight which children naturally bring to their learning. When this kind of teaching operates, mastery of number facts is accomplished with less "drill." Drill often means practicing errors and so does infinitely more harm than good.

The teacher who is himself in love with his subject has the best start in the world.

EDITOR'S NOTE. "It is the teacher's job to supply the experience necessary to invest symbols with real meaning for the students." It is so easy to mistake verbalization for understanding. Our job is to build understanding so that the verbal expression thereof has meaning for the child. This takes time and requires of the teacher a good deal of insight into the experiences and behavior patterns of children. Always, we must remember that it is the child who is being educated and that real education is a process within each individual learner.

Teacher's Dilemma

Many teachers have doubtless told their students that mathematics teaches truth and honesty, etc. Picture the dilemma of the teacher who could possibly be confronted with the following situation.

"You say teacher, that arithmetic is a science of truth," said Johnny. (This Johnny can read.)

"Yes," said teacher, "why do you question it?"

"You know," continued the teacher, "figures don't lie."

"Show me an example," said Johnny.

"Here is one, Johnny. One man can build a house in 20 days. Then 20 men should be able to build it in one day."

"But teacher," said Johnny, "then 1200 men should be able to build it in one minute; and 72,000 men should be able to do it in one second. I don't believe that they could drive one nail in that space of time. So where is the truth in arithmetic?"

While teacher was catching her breath from that onslaught of the precocious youngster, he started out with another series.

"If one jet airplane can fly around the earth in 24 hours, 24 airplanes should fly around it in one hour; 1440 in one minute, and 86,400 in one second. I don't believe that either."

Teacher was not yet over the first shock when the youngster began, "a pound of feathers is heavier than a pound of gold, and I don't believe that either."

Obviously the youngster had gone beyond his depth and teacher was able to recover from a situation that at first seemed out of hand.

Kindergartners Learn Arithmetic

Dorothy Campbell
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It has been said that "kindergarten is the bridge between home and school." The fact that learning is a continuous process that is based on all previous experience makes this statement a very significant one for the kindergarten teacher. When children enter kindergarten, they come with their specific backgrounds of concepts, of language and of skills. For all children, this includes mathematical experiencing. Children have been surrounded by quantitative relationships from their earliest experiencing. In kindergarten the concepts already learned can be further amplified and new understandings developed to form a firm foundation for mathematical experiencing throughout the school years and beyond.

It is relatively simple for us to teach kindergarten children number. The child is surrounded by number and his interest is easily stimulated. There are so many techniques to use to make number meaningful at this level that we are prone to overlook the even more basic "mathematicking" that can and should take place in kindergarten. Before we can do a good job of teaching quantitative relationships, we must develop an awareness of the many challenging experiences that come to us over the bridge from home to kindergarten.

"I didn't get a turn to paint," can lead to an understanding that there are more children than painting easels, and an understanding of this quantitative relationship leads to another kindergarten goal, learning to be part of a large group.

In kindergarten there is a fertile field for teaching the vocabulary of mathematics. A child who picks up a triangle shaped block and says, "This looks like a sandwich," opens the way for learning that the block that looks like a sandwich is shaped like a triangle. Two of these triangular blocks of the same size placed together become a square shape. "Oh look, here is a block that looks like a piece of cheese," may lead to the idea that this shape is called a wedge and that two of these wedges of the same size placed together become a shape called a rectangle. There are endless possibilities for incidental teaching gleaned from a remark dropped by a child and picked up by a teacher with awareness. "School was a long time today," may lead to a discussion of time and an opportunity to introduce vocabulary. However, much of our teaching should be planned. Our teaching tools need not be extravagant. They can come out of the home where previous experiences have made them familiar—cups, jars, boxes and cartons, spoons and cans. We use them in school also, in the doll corner and for mixing paint. A quart jar holds more water than a pint jar. A tablespoon holds more than a teaspoon. Let's measure and see how many teaspoons of water it will take to fill one tablespoon. Measuring the ingredients for a batch of cookies is a beginning for understanding fractional parts. Building blocks, standard equipment for kindergarten, are excellent tools. We see fractional parts here as well as shape and size. A kindergartner may bring a bag full of autumn leaves to school that he has collected while helping rake leaves in the yard. The teacher may use them for teaching tools in science and in art, and she may also use them in developing a concept of weight. The children may feel how heavy the leaves are by lifting the bag. Then they may feel the weight of another bag filled with something familiar to them, some crayons for example. They may estimate which of the two bags is the heavier, and, if there is one available, a balance

scale may be used to validate the estimate.

To develop understanding of distance, direction and location, and to develop a meaningful vocabulary in this area, a trip throughout the school may be planned. Then a walk about the neighborhood can be used to further these understandings. A teacher overheard some of the children discussing the fact that they could see more things from the top of the climbing pole in the room than they could see from the floor. The teacher made a note of this and used this experience in a planned situation at a later date. By doing this, the entire class had an opportunity to participate. The children discussed what they saw and how it looked from the top of the pole. Then they went outside to see what they could see from the ground and how it looked. The observation made by the whole class was that you could see more things from the top of the pole. When asked why, nearly all of the children volunteered that it was because they could look over near objects. This experience proved very valuable when the class made a mural map of the area around the school in culminating a unit on the home. They were able to understand direction, distance and location of their homes in a very real way.

The sharing period is one of the most opportune times of the kindergarten day for learning in all areas—science, art, geographical concepts, music,—to name a few of the academic areas and, of course, social adjustment. It is impossible to separate mathematical experiences from other areas of life. So, out of the sharing period can come a great deal of learning about quantitative relationships.

"We went on a trip yesterday," says Gail in sharing a weekend experience. "Where did you go?" the class wanted to know. "I'm not sure, but it was a long ride in the car," answered Gail. "What did you see? Maybe we can tell where you went from that," Bobby suggested. Gail answered that she saw cows and horses. The class decided that she must have passed farms or ranches. "We saw some houses," added Gail. "Were they close together or far apart?" Bobby asked. "They got closer together while we were riding," Gail answered. "Maybe she went to a city," Philip suggested. The teacher asked Gail if she and her family had gone to Los Angeles. Gail answered, "yes." "How far is that?", asked Gordon. The teacher explained that it is about 120 miles from El Cajon to Los Angeles. "Wow! that is a long way," several of the children exclaimed. This sharing experience developed geographical concepts as well as quantitative relationships. In another sharing experience Gary told about going aboard a submarine. When asked what he liked best he said, "I liked the tornados best." Gordon was quick to explain that Gary meant torpedo because "a tornado is a big storm with lots of wind that comes from the sky and does lots of damage." Gary did a very good job of estimating how long the torpedo was and he estimated how big around it was by making a circle with his arms. In explaining the shape he said it was "long, round, and fat." Philip volunteered that a torpedo is "long round and fat—but it is pointed on one end and has fins on the other." Because there are limitations on time in the sharing period, all of the opportunities for teaching can not and should not be used right on the spot. However, the teacher who is listening and watching for these opportunities can jot them down and use them in planning learning situations in the future.

The bulletin board is another valuable medium for developing mathematical concepts along with stimulating interest in language arts and art experiences. Numbers become more meaningful when a familiar story, poem for finger play such as 'The Three Bears' or 'Five Little Pumpkins,' is illustrated on the large bulletin board found in nearly every kindergarten room. The teacher does not need to limit the concepts to be developed to purely mathematical experiences. The beginnings of an understanding of perspective can make a child's art experiences more pleasant to him and more meaningful to older children and adults around him. It is very frustrating for a child

to see things with his eyes in perspective and to be unable to reproduce what he sees with a paint brush. It is at kindergarten age that children begin to have a desire to reproduce what they see around them with some degree of accuracy. Certainly in later life an engineer or architect will need to understand the "science of perspective."

While developing a unit on Indians and Pilgrims at Thanksgiving, a teacher arranged three Indian tepees on a large bulletin board with a background of hills, sky and a tree. The tepees were large, medium, and small. The large one was placed toward the bottom of the board, the medium one further up on the board and the small one just below the horizon. The first reaction the children had to the scene was that the father Indians lived in the big tent, the mother Indians lived in the middle sized tent and the baby Indians in the little tent. Then someone suggested that this wasn't the way families usually lived and the babies would be all alone. The teacher suggested that the children look out of the windows and talk about the sizes of things. One child spoke up "I know, that little tent is not for babies; it is *farther* away than the others." The other children began to see the picture in perspective also. Several days later a little girl was drawing a picture with crayons and asked the teacher to come and look at her picture. She explained, "This is going to fool you. That *little tiny* tree I made on top of the hill isn't really tiny. It's just *far away*." In developing this concept the teacher did not go into a detailed explanation of why it is true that things seem to be smaller as we get farther away from them. However, she and the children did establish the idea that our eyes see things this way. It will be a long time before these children will put the 'science of perspective' into practical use, except in their art experiences, but the basic concept has been introduced and understanding seems evident.

Growth in learning is a continuous process and we, as teachers, must keep uppermost in our minds that all growth is continuous. Children will differ in their ability to understand new concepts. We must be aware, not only of the possibilities for teaching, but we must also be aware of the readiness for learning of the class as a group, and of each child as an individual. The teacher who sees relative shapes and sizes as well as prettiness in the bouquet of flowers brought to her by a child, can soon establish the readiness of that child just through a friendly conversation. The teacher who hears fractional parts, shape, size and need for vocabulary in "My graham cracker looks like the windows in our room except that there are two parts to the cracker and four parts to the window," will recognize readiness of her learners and plan her program to meet the need. Growth is a continuous process for children and adults alike. We as teachers should be growing in awareness and sensitivity to the possibilities for teaching that surround us in our every day living in the kindergarten. And, by taking advantage of these possibilities our teaching can become more meaningful to the children and more rewarding to us.

EDITOR'S NOTE. Yes, there are many opportunities to learn about size, number, shape, direction, dimension, use, etc. in the kindergarten and many of these learnings are not immediately associated with number symbols. This is an arithmetic of concepts and ideas, of thinking and discovery. But it is not haphazard nor is it totally incidental. The good kindergarten teacher will see that all of her pupils have opportunity to experience, to visualize, to see, and to think and to develop ideas of size, shape, space, and quantity. These early concepts are very important to a child even though they are not precise and expressed in adult language. The art of being a good kindergarten teacher lies in the way experiences of children are conducted and capitalized upon. One teacher will *tell* the children what to do while her artist neighbor will help them to *discover* what to do.

Manipulative Materials in Intermediate Grades

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NOW THAT SO MANY of our elementary teachers are well versed in child growth and in the over-all pattern of a developmental curriculum, they are increasingly aware of the need for providing a stimulating environment that will intrigue the child and evoke a desire to try, to achieve, to learn, and to share. Greater attention, therefore, is given today toward making available multi-sensory aids for learning. It is evident that there is a growing awareness of the values of manipulative materials as an aid in teaching arithmetic.

Where an interest corner is set aside for reading, science, or math, there pupils tend to congregate, experiment, and discuss ideas. The child who reads an exciting book recommends it to his pal; the one who discovers that a floating candle will continue to burn for some seconds after a glass is inverted over it, is faced by questions that impel consultation and research. The continual cycle of handling, trying out, discovering, sharing, and exchanging observations leads from what is purely mainpulative into the realm of ideas—and *it is with ideas that we think.*

The possibilities of an arithmetic interest corner are limitless. Displays can be very simple and inexpensive—or just the opposite. The materials displayed should be changed frequently even though they may re-appear later for the element of novelty should not be overlooked. It is wiser, too, I think to have a few materials exhibited with real salesmanship than to have many cluttering the table and tending to repulse the orderly mind or confuse the shy child. Available on the arithmetic table at times should be copies of arithmetic books, possibly of previous and on-coming grades. I have seen children who were not particularly interested in any school subject, as such, pounce on a previous arithmetic text and joyfully prove they could work "the hardest problem in it!" Why is it, I wonder, that we adults forget too often the fun of reminiscing over books we used to study? Even the slowest child in a classroom can get some pleasure out of thumbing through a previous text. And today, when the arithmetic books are so colorful, so appealing in illustrations and make-up, and so carefully organized as to content level of difficulty, a child will like to peruse the familiar pages through which he may have struggled.

Use Textbooks for Reference

One teacher I know puts a copy of earlier arithmetic texts on display in the fall and places this sign on the table: DO IT YOURSELF and, in smaller print, she asks: WHAT CAN YOU MAKE THAT YOU LEARNED ABOUT LAST YEAR, OR THE YEAR BEFORE, OR THE YEAR BEFORE THAT? The pupils study the texts, form committees in terms of what they're going to build or make, choose the date for the exhibit, prepare labels and posters, place markers in the book to indicate the pages being illustrated, assemble additional research materials, and get to work. Discipline is no problem and drill work, as such, is not mentioned.

It is fascinating indeed to note the ingenuity of boys and girls when they are interested and become self-propelling. The pooling of their ideas provides a richer background by far than any the autocratic teacher can give regardless of her scholarship and persistence.

The use of manipulative materials is not an end in itself, nor is it a pastime for keeping pupils "out of mischief" while the teacher works with other groups. It is a vital, challenging phase of the learning process that gives impetus to participation in the problem solving experiences.

Problems with numbers and values which

pupils work out alone need not necessarily clarify the arithmetic they are learning in class. You may have heard a teacher say, for example, "They can't seem to add or count—yet look at them keep score during the games!" or "They can't do long division but they figure batting averages with ease!" It is easy for an adult to see the connection but quite often good teaching is essential to help the child bridge the gap. When the child senses the similarities, he learns with amazing ease. An old Vermont story comes to mind. It was in the days when ice cream cones cost a nickel. The teacher asked: "How many ice cream cones can I buy for a quarter?" Johnny volunteered to do the problem at the blackboard and proceeded to multiply 25×5. He turned to the teacher and proudly said, "You can get 125 ice cream cones."

"Oh well, if that's so, Johnny, here's twenty-five cents; go get us 125 cones!" said the teacher as she extended the coin.

Johnny made no motion to take the money; he looked at her in disgust as he said, "Heck, I thought this was arithmetic; I didn't think you meant it for *real!*"

One must realize that while experiences enrich the background and develop a readiness, the teacher is a vital factor in utilizing the knowledge gained so that it is meaningful to the learner.

Let us consider some of the opportunities for handling objects that may make arithmetic more meaningful in the intermediate grades.

Equipment inviting experimentation in computing and measuring

Scales (both types)
Liquid measures
Thermometers
Rulers
Tuning fork
Levers
Pulleys
Perpetual calendars
Measuring cups and spoons
Directional compass
Compass for drawing circles
Pedometer
Map meter
Stop Watch
Weights
Clocks
Flannel board & illustrations (fractional parts, magic square numbers, etc.)
Erector sets
Balloons
Varied lengths of pipes
Varied sizes of glasses
Barometer
Egg cartons
Hour glass
Sun dials
Tape measures
Metronome
Toy piano
Pitch pipe
Catalogues
Toy money
Chess
Dominoes
Monopoly (game)
Globe, maps

Experiences which involve number, counting, computing or measuring

Physical Education:
- Marching by 2's, 3's, 4's
- Rhythmic counts
- Square dancing
- Keeping score
- Setting dimensions for court or field
- Keeping time for relays
- Number ball toss (Two teams with members numbered; when number is called, child must catch tossed ball before it touches ground to score a point for team.)

Music
- Arranging varied lengths of pipe so tune can be played
- Filling glasses with water enough so scale can be played
- Raising or lowering of pitch by tightening or loosening a stretched wire
- Locating pitch of sound made by tapping glass, block of wood, piece of metal, etc.

Art
- Enlarging by means of squares a small picture (or vice versa)
- Design a movable color wheel so that as you spin parts, each section on top is smaller than one below; label fractional parts.

Arithmetic
- Design a multiplication wheel for the lower grades (or a division wheel)
- Make personal playing cards with additional facts on them; put answers on back. Work with a pal who will flash the cards.
- What 2 objects here equal the weight of three others? Use the scales to check your guess.
- Cut the numbers one to nine from a large calendar. Arrange them in three even columns so that they equal fifteen horizontally, vertically, and diagonally.
- Use colored spools, beads, and string to make an abacus.

Social Studies, etc.
- Record daily weather data

Keep records of food fed hamsters during nutrition experiment; chart weight changes.

Clocks and maps. If every 15 degree equals one hour of time, set the clock to the time it is now in London, San Francisco, Chicago.

To continue the list is unnecessary for it becomes longer as one continues to sit and ponder. That shared experiences with such multi-sensory aids will enrich the children's background and help raise the level of attainment in arithmetic, I know to be true.

EDITOR'S NOTE. The idea of an arithmetic corner of the classroom sounds very good especially if it contains interesting and useful materials. Textbooks of a year or two below the current grade are most useful for relearning and this usually can be done with a minimum of instruction from the teacher. It is a worthwhile aim for children to learn to use a textbook as a reference and the ability to learn independently from a book has continuing value. Miss Fox suggests using arithmetic in connection with other areas of learning such as music and social studies and she shows that there can be real pleasure in learning arithmetic.

Item on Reading Decimal Fractions

Can you read the following or write them in words so that there is no mistaking the amount expressed?

A. $2.0\frac{2}{3}$ B. $0.2\frac{2}{3}$

Many years ago, Kate Savage, a teacher, carefully explained to her pupils that any fractional amount less than a whole takes the singular form. She also explained that any common fraction has no home of its own (place in number system) but is a tail attached to the previous place. In terms of her explanation, exercise A. should be read "two and two-thirds (of a) tenth" and B. as "two and two-thirds tenths."

It is commonly said that the common fraction $\frac{1}{2}$ when it appears in a decimal can be changed to a figure 5. For example, $0.7\frac{1}{2}$ becomes 0.75 and $0.00\frac{1}{2}$ becomes 0.005. But what shall we do with $0.\frac{1}{2}$? Is this 0.5? But $\frac{1}{2}$ without the point is 0.5. By Kate Savage's reasoning, $0.\frac{1}{2}$ is attached to the ones' place. The decimal point does not occupy a place in our system.

Filmstrips Reviewed

Arithmetic Series—Set No. 1. Young America Films, Inc., 18 East 41st Street, New York 17, New York, 1956.

(1) What Are Numbers
(2) Whole Numbers
(3) Addition and Subtraction (Numbers to 10)
(4) Addition and Subtraction (Numbers beyond 10)
(5) Multiplication and Division
(6) Solving Problems

Respectively, these filmstrips bring out (1) the cardinal and ordinal concept of number, the study of groups, and the concept of measurement; (2) reading and writing whole numbers; (3) idea of putting groups together, the commutative and associative laws of addition, odd and even numbers, and problem solving; (4) addition and subtraction and our system of numeration; (5) the concept of multiplication and division, the commutative, associative, and distributive laws of multiplication, and these operations using our system of numeration; and (6) 9 basic questions that might be used in solving problems, illustrating the use of these questions by solving three selected problems.

Although there are some very good ideas in each filmstrip, the reviewer recommends only #1 and #3 to be used as supplementary materials in building arithmetical concepts in the classroom. There are some good ideas in #2 and #4, but the sequential development detracts from the effective use of the filmstrip in its entirety. Selection of specific frames desirable is difficult. The reviewer sees little, if any, contribution that #5 and #6 might make in the classroom. Yet, a creative teacher may find any of the last four of these filmstrips usable as a means of stimulating questions regarding ideas of fundamental operations and ways of solving problems. As materials for preservice and inservice teacher education, an instructor may find use for them as a means for critically examining possible instructional practices in the classroom.

E. GLENADINE GIBB

An Aid in the Analysis of Verbal Problems

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The problem of teaching the solution of verbal problems in arithmetic has for many years been an intriguing one. Just about all points of view have appeared in the literature at one time or another. Yet today the question of what is best is still a very open one.

The current prevailing point of view would seem to be one of permitting the student to develop his own method for solving problems. Nevertheless, contrary to what some authors[1] of this school of thought maintain, there may still be something to be gained from discussing verbal problems as a topic and from analyzing their solution systematically.

Most students have developed methods for solving verbal problems long before they reach junior high school, but for years teachers have observed that their methods so very often do not hold up when it comes to the solution of more difficult problems. It would seem that the "helter-skelter" approach which some students have previously found to be successful must at some point be re-evaluated and often replaced by a systematic approach to the concrete problems of elementary and junior high school arithmetic which can also later serve as a foundation for the more abstract forms of thinking required in algebra and geometry. We, as teachers, cannot teach students precisely how to think, but we do have a responsibility to help develop within students methods for organizing their thoughts for problems of increasing difficulty. No mechanical method can be a cure-all for all students. However, a mechanical method taught as a topic can be an excellent frame of reference upon which students can base their arithmetical thinking.

The procedure which is to be presented here is an attempt to build such a base and to find out what students are thinking as they reason out problems by having them put some of their thoughts down on paper.[2]

In every verbal problem there are certain *given facts* involving numerical quantities which the student must be aware if he is to solve the problem correctly. Given these facts the student is asked *to find* the answer to a certain question. No matter what type of a thought process the student is using he must either consciously or unconsciously record and store (at least temporarily) this essential information. In using this procedure the student is asked to write on paper what he is recording in his mind, so that his thoughts can be noted. He is asked to do this in a certain way—which is written out in detail below.

Once this information has been recorded and stored the student must then begin to think Again, no matter what process of thought the student may be using, it would seem that if he is to arrive at a correct solution he must decide on what *hidden questions* (or, if you prefer, *sub-problems*) must be answered before the main question (i.e., the *to find*) can be arrived at. In order that his method may be observed and diagnosed the student is again asked to put his thoughts down on paper.

When the student has decided what *hidden questions* he must solve he is then ready to develop a *method* for answering them and then finally for answering the *to find*. This he is also asked to put down on paper.

To visualize this procedure better, let us go back through it with the following problem in mind.

[1] See, for example: Peeler, Harry, "Teaching Verbal Problems in Arithmetic," The Arithmetic Teacher, Dec., 1956, 244–46.

[2] The procedure discussed below is an extension of the Three Column Form procedure. See: Jackson, Humphrey, "Method—Computation—Answer," *The Mathematics Teacher*, Oct., 1956, 492–93.

PROBLEM: Jane's mother gave her a $1 bill and asked her to go to the store to get 6 cans of tomato soup. If tomato soup is selling at 2 cans for 27¢ how much change will Jane receive?

In the "REASONING column" of his paper the student would be expected to record his thoughts along lines similar to the following.

Given:

1. Mother gave Jane a $1 bill
2. Jane is to buy 6 cans of soup
3. Soup is selling at 2 cans for 27¢

To Find:

1. How much change will Jane receive?

Hidden Question:

1. How many groups of 2 cans is she buying?
2. How much will all these groups cost?

Method:

1. Divide: 6 cans by 2 cans
2. Multiply: 27¢ by the answer from step 1
3. Subtract: from $1 the answer to step 2

What has been described above involves considerable writing on the part of the student. The students realize this very quickly and they generally waste no time in protesting. In reply to any protests, the teacher need only point out that not being a mind reader he has no way of knowing what is going on inside each of their individual minds unless they bring it out into the open where it can be observed. It would seem that the best way to do this with an entire class is to have them write their thoughts down on paper.

The students should also be informed that once this method has been developed as something to fall back on, and once the teacher has been able to diagnose and attempt, as best he can, to correct faulty practices, they may eliminate the writing out of those parts of the procedure which they honestly feel—and can effectively demonstrate—are no longer necessary. This introduces the necessary freedom and consideration for individual differences, but yet leaves the slow and often insecure student with something to fall back on when the going gets a bit rougher later on. And it is always interesting for the teacher to note how the students who at first protested the loudest at the idea of having to do "all this writing," are so often the ones who—finding the procedure getting them consistently correct answers—want to keep it the longest.

Once the products of their thoughts are written down on paper the students are then ready to do their computations in the COMPUTATION column. These computations—if they are to be analyzed for errors—must be written down and numbered in an orderly fashion. Then, when the answer is arrived at it is written in the ANSWER column.

The entire problem would then look something like this.

REASONING	COMPUTATION	ANS.
Given: 1. Mother gave Jane a $1 bill 2. Jane is to buy 6 cans of soup 3. Soup is selling at 2 cans for 27¢	① $2\overline{)6}$ quotient 3	
To Find: 1. How much change will Jane receive?	② $.27 × 3 = $.81	
Hidden Questions: 1. How many groups of 2 cans is she buying? 2. How much will all these groups cost?	③ $1.00 − .81 = $ .19	$.19
Method: 1. Divide: 6 cans by 2 cans 2. Multiply: 27¢ by 3 3. Subtract: 81¢ from $1		

This procedure can be a laborious and stifling process for the student who is quick to grasp abstractions. Because of this, once the original teaching phase—if this procedure is being used in the elementary grades—or the diagnostic phase—if it is being used in the junior high school—is over, each student must be weaned away from those steps or aspects of the procedure which for him are no longer necessary.

The writing down of the *given facts* can be eliminated very quickly, then the *to find*, and then the *hidden questions*. One must be careful in eliminating the writing out of the *method*, for having to write this out is often of great help in getting the impetuous and careless student to think through what he is doing and should in these cases be retained long after a student has demonstrated his ability to reason systematically, if that student is not consistently getting correct answers.

Finally the writing out of all of the computations can be eliminated if the student can do them accurately in his head. When this point is reached it would seem that one can begin to rejoice. If everything but the writing down of the answer is being done mentally with accuracy and speed, for what more could we ask?

However, if at any time a student begins to become inaccurate or is finding it difficult to carry all of his thinking in his head he should be encouraged—and even required—to fall back on this frame of reference until he, or the teacher, has made the necessary adjustments or corrections.

In conclusion, it should be stated again that this procedure is not being presented as a panacea for all verbal reasoning ills. It has, however, demonstrated its worth in practice, and to a degree sufficient enough to warrant its being shared with others. We, as arithmetic teachers, cannot teach students *thinking, per se*, but we can, if we are alert, provide some of the necessary conditions under which careful thinking can be encouraged, observed, and evaluated.

EDITOR'S NOTE. Mr. Herriott has used this method for several years and with good success. Note that he is not one who insists that a formal method be retained and used long after it is no longer necessary. But he is interested in organized and disciplined thinking and for many youngsters this is a needed step in "growing up" in arithmetic. After all, we hold schools so that children may learn and it is our job as teachers to help them to learn and occasionally this help must take the form of compulsion. The ability to think coupled with the will to think is one of our larger aims in arithmetic. Problem solving is one of the avenues through which pupils learn to think.

A Method in Division of Whole Numbers

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WHEN STUDYING THE PROBLEMS which arise in our environment, we find that there are two distinct types which call for the division of whole numbers in their solution. They are

a. finding how many groups there are in a whole when the size of one of the equal groups is given
b. Finding the size of one group when the whole and the number of equal groups into which it is to be divided are given

Now, as the title of this presentation indicates, is there one generalized procedure by which both kinds of problems may be solved? The answer to this question is "yes" if we

gather the import of what B. R. Buckingham[1] has written. He has told us in essence that, although there be two kinds of division problems, there is numerically but one operation.

If there is but one operation, and it is to portray the generalized concept in the division of whole numbers, even though there are two kinds of problems, what is it, and what are the meanings involved? Dr. Buckingham in his treatise has not given us the development as a classroom learning-concept. Therefore it becomes the purpose of this presentation to describe how it may be done.

The readiness phase. It is important that we give attention to the preparatory work which we feel is necessary in the solution of our problem. As we see it, we should present the developments of the concepts within the two types of problems. We shall build upon that which Professor H. C. Christofferson has so excellently described for us in a former issue of this journal.[2] We shall go into detail and present what we think the Professor would do in a classroom development in order to achieve the learnings as he has stated them. We shall use different problem situations, however. Our two problems are

(a) Partition in division:

How many oranges are there in one group when a group of 84 oranges is divided equally into 6 groups?

(b) Measurement in division:

How many 6¢ stamps may be purchased with a total of 84¢?

The developments are as follows:

(a) Divide a group of 84 oranges into 6 equal groups;

$$6\overline{)84 \text{ oranges}} = ?$$

By the counting phase, we have two steps

(1) 0000000000 . . . , 84 ones distributed one at a time in each of 6 boxes until all are used;

00···	00···	00···	00···	00···	00···
·····	·····	·····	·····	·····	·····
·····	·····	·····	·····	·····	·····

When the number in each box is counted, it will be found that there are 14 oranges in each group.

(2) Using tens and ones counters to represent the 84 oranges,

84 ones = 8 tens and 4 ones = 6 tens and 24 ones

8 tens + 4 ones

O O O O O O O O + 0 0 0 0

or

6 tens + 24 ones

O O O O O O + 000000000000000000000000

distributed one at a time in the 6 boxes,

[1] B. R. Buckingham. *Elementary Arithmetic—Its Meaning and Practice.* (Ginn and Company, Boston, 1953) p. 76.

[2] H. C. Christofferson, "Meanings in Division," THE ARITHMETIC TEACHER, February, 1957, pp. 21–23.

O 0000	O 0000	O 0000	O 0000	O 0000	O 0000

and we have 1 ten and 4 ones in each box, or 14 oranges.

The next steps are the computational phases (abstract, and based upon (2) in the counting phase)

First, we write the place value names,

$$6\overline{)84} = 6\overline{)8 \text{ tens and } 4 \text{ ones}} = 6\overline{)6 \text{ tens and } 24 \text{ ones}}$$

from which we obtain 1 ten and 4 ones, or 14 oranges. Then, by thinking the place values,

$$\begin{array}{r} 14 \\ 6\overline{)84} \\ \underline{6} \\ 24 \\ \underline{24} \end{array}$$

6 tens divided into 6 equal parts; we have 1 ten in each part for the first quotient figure.

24 ones divided into 6 equal parts; we have 4 ones in each part for the second figure

1 ten and 4 ones = 14, or 14 oranges.

(b) How many 6¢ stamps may be bought for 84¢? 84¢÷6¢=? By the counting phase, we have

0000000000 . . . , 84 ones separated into groups of 6 ones, 000000, 000000, and so on. Thus when all of the 84 are used, the number of groups is found to be 14. The answer is 14 stamps.

Now it is obvious that tens and ones counters are not usable without a regrouping, hence 84 = 6 tens and 24 ones. But, as a first step, even the 6 tens and 24 ones divided by 6 ones is complex. So, to give it meaning, we should consider a transformation which is more simple, namely (in the abstract),

$$6 \text{ ones}\overline{)6 \text{ tens and } 24 \text{ ones}} = 6 \text{ ones}\overline{)60 \text{ ones and } 24 \text{ ones}},$$

whereby the values 10 and 4 are obtained, or a total of 14 stamps. Then, it is clear that

$$6\overline{)84} = 6 \text{ ones}\overline{)6 \text{ tens and } 24 \text{ ones}} = 1 \text{ ten and } 4 \text{ ones, or } 14,$$

whence the following forms in division are understood.

$\begin{array}{r} 4 \\ \underline{10}\ \rangle 14 \\ 6\overline{)84} \\ \underline{60} \\ 24 \\ \underline{24} \end{array}$	60 ones are usable to give a tens value. How many groups of 6 ones are there in 60 ones? The answer is 10. Etc.	$\begin{array}{r} 14 \\ 6\overline{)84} \\ \underline{6} \\ 24 \\ \underline{24} \end{array}$	6 tens are usable. How many 6's are there in 6 tens? The answer is 1 ten. Etc.

Therefore we see the Professor's deductions, as applied to (a) and (b), to be the following; 84÷6 can mean two different relationships: (1) if 84 is divided into 6 equal parts, there will be 14 in each equal part, and (2) how many 6's can be subtracted from 84, or separated from it?

The One Method in Division of Whole Numbers

We shall now proceed to show that we can generalize the "How many 6's are there in 84?" whether we are dividing the whole into 6 equal parts or finding how many groups of 6 there are in it.

Measurement was the process of finding how many 6's, so we shall show that *Partition* may be so considered.

)84 oranges—dividing the whole into 6 equal parts

We shall use boxes in the counting phase as before.

84 oranges distributed →

?	?	?	?	?	?

Now, when one orange is placed in each of the boxes, we have one in each of the 6 equal parts to be determined. In this process

0	0	0	0	0	0

we have used a group of 6 oranges out of the total. Again, if another orange is put into each box, we have this,

00	00	00	00	00	00

which shows that another group of 6 was taken from those remaining. Therefore it is seen that the number of oranges in each box (or in each part) will be as many as there are 6's in the total of 84. And, accordingly, we have changed the method of partition into one of measurement. Thus, in abstraction, the thinking would be precisely what it was in the development that was identified previously as, How many 6's are there in 84?

Finally we must say that maturity plays a part in the child's ability to achieve the meaning of this *one method* for doing both types of division problems. The process belongs in the fifth grade work, assuming that readiness has been established by means of introductory-preparatory teaching in the nature of the concepts as presented by Professor Christofferson.

EDITOR'S NOTE. Mrs. Adams begins with "measurement" and "partition" as two aspects of division situations and then shows how these can be combined into a single concept of division. It would be a hindrance if we had to stop and consider each division situation in terms of the two aspects before we started to divide. Many teachers consider this an essential step in the development. Certainly we learn the division process in order to use it in problem situations. However, when we finally become good dividers we are operators with abstract numbers. Probably the most common uses of division are represented in the situations associated with the generalizations $C=np$ and $D=rt$. But there is a considerable step from the first concept of division to the final abstract work with numbers and Mrs. Wood has given a rational development which children can understand.

Note on the Teaching of "Ragged Decimals"

HARRY E. BENZ
Ohio University, Athens

THE PURPOSE OF THIS PAPER is to raise certain questions regarding the almost unanimous condemnation of the teaching of the addition of decimal fractions with the aid of practice material which contains examples in which not all the addends contain the same number of digits on the right of the decimal point. Such addition examples are often referred to as "ragged decimals." In this discussion we shall overlook the fact that it is not the numbers themselves, but the columns which are "ragged," and we shall use the commonly accepted term to refer to such examples.

Most recent writers on the teaching of arithmetic protest the use of such examples, and some of them condemn such usage in language which seems somewhat dogmatic. Quotations from several such writers follow. Brueckner and Grossnickle say, "Examples of this kind never represent a socially significant situation. . . . Ragged decimals never occur in social usage."[1] Morton says, "This kind of example should not be given, for it does not occur in ordinary uses of decimals."[2]

After referring to what is accepted measurement practice, Spitzer says, "There is then no good reason for giving elementary-school children practice in adding ragged decimals."[3] Clark and Eads are somewhat less certain in their statement on this point. They say that the situation under discussion "seldom occurs in practical experience situations."[4]

Several of the writers who discuss this situation go into some detail in justifying their positions. In general, the argument may be summarized as follows: In concrete situations, the numbers added represent measurements. Measurements in a given situation are, or ought to be, always expressed to the same degree of precision. The addends in a given problem will then all have the same number of decimal places. Therefore, addition examples which involve addends with varying numbers of decimal places represent an artificial situation which would never occur in real life.

Notice that the case presented rests on the question of the social utility of the skill in question. The ability to add ragged decimals is not needed in the affairs of everyday life. Therefore it need not be taught. Note some assumptions here. It is assumed that in all instances in which a person has occasion to add decimals, the numbers will be the results of measurements. It is assumed that such measurements will always be expressed in the manner that good measurement theory suggests, namely to the same number of decimal places.

Careful search of the literature reveals little information which will answer the question, "In what situations do people need to add decimals in everyday life, and what kind of numbers are they called upon to add?" The assumption that all such situations will involve measurement may be sound, but it should be questioned. And the assumption that such measurements will always be made by persons with sufficient understanding of the nature of measurement to express their results correctly, may also be questioned. It seems that before we can be too certain that ragged decimals do not occur in the affairs of everyday life, much more careful study of the arithmetical

[1] Brueckner, L. J., and Grossnickle, F. E., *Making Arithmetic Meaningful*, John C. Winston Co., Philadelphia, 1953, p. 406.

[2] Morton, R. L., *Teaching Children Arithmetic*, Silver Burdett, New York, 1953, p. 327.

[3] Spitzer, H. F., *Teaching of Arithmetic*, Houghton Mifflin Co., Boston, 1954, p. 248.

[4] Clark, John R., and Eads, Laura K., *Guiding Arithmetic Learning*, World Book Co., New York, 1954, p. 193.

needs of people will have to be made. A specific instance is here presented for what it may be worth. The reader is cautioned not to accept the report here given as a piece of "research" in the social uses of decimals, but merely a report of an episode.

Not long ago the writer made an automobile trip to the Pacific coast and back, buying his gasoline along the road with the aid of a "credit card." Upon his return he determined to satisfy his curiosity relative to the perennial question, "how many miles per gallon?" The accumulated duplicate sales slips were examined, and the numbers representing the numbers of gallons purchased were set in a column to be added. It should be said that this writer, when purchasing gasoline, sometimes says, "Put in *x* gallons," and sometimes says, "Fill it up." Obviously, an 8300 mile trip involved a large number of stops for fuel, and only a part of the column of numbers needs to be shown in order to make the point. The example at the right represents a fragment of the column.

```
11.4
15.0
12.2
16.
14.35
11.0
10.
12.7
 8.5
10.
```

It seems apparent that service station attendants either receive varying instructions about how to make out sales slips, or vary in their devotion to their instructions. Probably both are true. In any case, not all the measurements shown here are expressed with the same precision. One may speculate on how one figure happened to be expressed in hundredths, when the dials on pumps are not scaled to that degree of accuracy. A good guess would be that the automatic cut-off on the hose stopped the flow at a point where the dial registered about halfway between 14.3 and 14.4, and the operator, wanting to convey the proper information about the amount, wrote 14.35. No computation for cost is necessary, since all pumps compute the total cost automatically and the attendant simply copies the figures.

The fact remains that the writer was faced with a socially useful situation in which he had to add "ragged decimals." Having at one time received instruction in this form of example, he was able to meet the situation.

We lack information on the important question of just what uses ordinary people make of the skill, addition of decimals. We know that they have many occasions to add numbers which represent sums of money. These do not usually result in "ragged decimals." Perhaps they do not add enough other decimals to justify teaching the skill at all. Nobody knows. However, it does not seem likely that the issue can be settled by reference to what the numbers would be like if everyone followed good measurement theory. Further investigation of the social uses of the skill is needed.

Another matter might well be investigated. It is now generally agreed that meaning should be emphasized in arithmetic teaching. The whole question of meaning has been so extensively discussed in recent years that one may assume that the reader understands what is meant. One aspect of the matter has not often been as explicitly stated as it deserves. While arithmetic may be taught primarily because it is needed in the everyday affairs of people, it also represents one of the great intellectual achievements of the human race. A knowledge of the nature of number, the rationale of arithmetical processes, and an understanding of the number system are part of the cultural equipment of an educated person. If this is true there might be some topics in arithmetic, or some variations in arithmetical processes which could be justified on the ground that they illuminate the pupil's understanding of the number system.

It is just possible that some work in adding "ragged decimals," with careful explanation from the teacher about the meaning of the empty spaces, would contribute to an understanding of place value. Children are taught the importance of zero as a place holder. Zero also has other meanings and other functions, and one of those functions is to indicate something about the degree of precision with which a measurement was made. When "ragged decimals" are taught,

some teachers permit pupils to fill the vacant spaces along the right with zeros. This presumably is to increase accuracy by helping the pupil keep his columns straight as he adds. Are these zeros place-holders?

One other instance of adding "ragged decimals" should be mentioned, although the presence of situations such as those about to be described would hardly justify the kind of drill in this disputed topic which characterized textbooks of fifty years ago. Many persons have occasion to add columns of numbers which represent sums of money, in which some of the amounts are represented as even dollars and some are expressed in dollars and cents. Most of us when faced with a situation calling for the addition of such numbers, more or less automatically put the two zeros in their places when we list the numbers for addition. But they do not always come to us in that form. The price tag says "$26." Nor does that mean twenty-six dollars to the nearest dollar; it means exactly twenty-six dollars. Just what is the individual doing when he puts the zeros in their places? Is he trying to record the fact that the cents were not overlooked, and that the price actually is twenty-six dollars and *no* cents, is he introducing a "crutch" to facilitate his addition, or is he conforming to some vague "convention" which governs the writing of sums of money?

$14.75
26.
9.48
13.27
17.

The present writer has no good answers to the questions raised here. However, it does seem that some more careful thought, and some research is still needed so that we may clarify our thinking on this whole matter of the place of "ragged decimals" in the arithmetic curriculum.

Editor's Note. Professor Benz finds that the standard reference books may be at variance with practitioners in the art of recording numbers. As Benz points out, a series of numbers representing measurements may be furnished by several different people who may be observing somewhat different procedures. He also points to the mathematical understanding that might be derived from working with "ragged decimals." Usually someone will find an exception to a man-made rule that is all inclusive in scope. Situations and circumstances differ. For example, the man who said that no one ever used decimal fractions of more than four digits should have said that he had never done so. Likewise the teacher of statistics who stressed "four significant figures" should not have been too alarmed when a student presented the standard error of the mean as 0.0300.

Pamphlet on Grades Seven and Eight

The Commission on Mathematics of the College Entrance Examination Board has issued a four-page pamphlet "The Mathematics of the Seventh and Eighth Grades." This is available from the Commission's office at 425 West 117 Street, New York 27, N. Y. Curriculum workers may be interested in this brief presentation which describes the arithmetic, geometry, and algebra suggested for these grades as basic for continued study of mathematics. While method is not suggested the ways in which certain parts of the content are stated might lead one to draw conclusions concerning method. For example, "Per cent as an application of ratio. The understanding of the language of per cent (rate), percentage, and base. In particular, the ability to find any one of these three designated numbers given the other two." This has a flavor of an algebraic approach when many teachers in these grades will prefer one more arithmetic derived from a study of per cent as it is used and the development of basic principles therefrom. In the relationships among geometric elements the idea of symmetry is mentioned but not those of equality and congruence. Few people will quarrel with the prescriptions of the committee in matters of content. This reviewer believes that the modes of learning might well have been discussed because these may be equally as important as the content and this may be especially significant for the more able pupils who will continue the study of mathematics.

Ben A. Sueltz

Directional Exercises as Preparation for Algebra

William G. Mehl
Pasadena, California

We who are teaching eighth grade arithmetic may find a stimulating method of aiding students in their preparation for algebra in a device which I have termed, *The Directional Exercise.* The material which might be incorporated into the drill is legion. It may be based entirely upon previous class experiences, or may relate to the introduction of new ones. Emphasis is placed upon implicit statements rather than upon explicit ones at this level.

Personal experience has pointed out a deficiency upon the part of the students in coping with the simple following of directions. This, together with insufficient drill material in problems dealing with common and decimal fractions and terminology, led to the creation of the instructional device shown in this article. It would seem that other teachers might, as did I, employ the device as a remedial technique toward stimulating concepts in the arithmetical processes with common and decimal fractions as their nucleus. Such would, of course, be optional with the teacher, as the device and its usage is not limited to any rigid element.

The implicit nature of the statements, coupled with a portion of the basic algebraic notation might serve as a stimulus toward the preparation for algebra. The scope of such material would be left to the discretion of the individual teacher. Students in the past have found such material to be refreshing.

The device may also be employed for review just prior to examinations. Students often express a desire to formulate several questions in the manner outlined to exchange with others during a review period.

Learn to Follow Directions

The following exercise is merely suggestive and is intended to illustrate the idea of the *directional device.* It is hoped that teachers would formulate similar exercises designed around material which meets with their student's particular needs and which emphasizes the particular teacher's creative ability.

Add 0.709 to:

1. .5+7.4+00.3+5
2. 6.0+.005+0.10
3. The average of the first two sums
4. The product of .5 and .60
5. The first addend in: 100.907+3.5+40.700+2
6. The quotient in: 6.060÷3.0
7. Your present age plus .6
8. The square of .7
9. The divisor in $\frac{80.962}{.03}$
10. The remainder in: .905−.45
11. (13.02+.2) (.8)
12. Itself less .3

Multiply three thousandths by:

13. One more than 3.9
14. Six times twelve
15. One-half dozen
16. The cube of three
17. Four less than 6.6
18. $\frac{(.21)(1.6)}{.4}$
19. The square root of 144
20. The average of: 72, 13, 50, 105, and 38

Subtract 3.201 from:

21. The digits in our current year
22. Eight and four-tenths
23. Six million and two
24. The dividend in: 4.8602÷4
25. The year Columbus discovered America
26. x when $2x$ equals ten
27. $(9)^2$ minus the square root of thirty-six
28. $3(10)^2+0(10)+5(1)$
29. The largest prime factor of 26

Arithmetic Problems: Cause and Solution

W. R. Taylor
Burroughs Corporation, Detroit, Mich.

There is considerable confusion among teachers and administrators about the steps that should be taken to meet the demands for more—and better—arithmetic instruction coming from industry, government, science and the general public.

There is little argument that a problem exists in the teaching and learning of arithmetic. If there is any doubt, it can be dispelled quickly by reading through the list of topics covered in the six issues of The Arithmetic Teacher for 1957.[1] Further proof would be the resolution adopted by the National Council of Teachers of Mathematics at its business meeting last March, which certainly recognized the existence of the problem.[2]

While the problem is recognized, there are many opinions about its basic causes. Educators, psychologists, sociologists, business executives, government leaders, and the public have all expressed their views and offered solutions to the problem without even suggesting a possible cause.

It is certainly arbitrary to accept a study implying only one cause. To do so, however, would eliminate the confusion resulting from ignoring cause and suggesting only a solution to the problem.

A study which discusses causes was made by the Educational Testing Service of Princeton, N. J., under a grant from the Carnegie Foundation. In the report, issued late in 1956, it was pointed out "complaints" exist:

> "That even the most elementary skill, facility in simple arithmetic, is in short supply."[3]

The researchers found that:

> "The average person does not get very far in mathematics. In the elementary school, he goes through the basic operations in arithmetic, but after entering high school, he tapers off fast. . . . Many of those who rate well above average in intelligence do not pursue the subject as far as they might. . . . The reasons given for abandoning mathematics are . . . they just don't like the stuff; they are afraid of it; they don't see the point of it."[4]

If an implication can be drawn from these findings, it would be that the students have lost interest in mathematics. Psychologists might tell us that interest is synonymous with understanding. It can therefore be assumed that there is a lack of interest because there is a lack of understanding. A lack of understanding of what? Basic fundamentals!

Taking a positive approach, student interest in arithmetic must be revived. To verify this factor of interest, Alice M. Hach of Racine, Wisconsin, says:

> "Add interest to junior high mathematics to whet appetites for senior high mathematics."[5]

But how, you ask? Especially significant, therefore, at this point, the researchers at Princeton recommend:

> ". . . stimulatlng and training teachers to try out new ideas in the classroom, studying the responses of students to see how the new ideas are taking hold, and measuring each step to see if the differences are worthwhile and permanent.[6]

This recommendation could be altered to read:

> ". . . to see how the new ideas revive student interest. . . ."

The writer, with a background of 20 years in the classroom and teaching of seventh grade mathematics, was faced with the problem that now has come into prominence. A solution, a new method, was sought and

[1] The Arithmetic Teacher, December 1957, pp. 275 ff. The classified index identifies 58 different articles on teaching arithmetic at all levels.

[2] The Arithmetic Teacher, November 1957, pp. 230 ff.

[3] Educational Testing Service Report: Problems of Mathematical Education, Princeton, N. J., 1956, p. 1.

[4] *Ibid.*, p. 3.

[5] *N.E.A. Journal*, October 1957, p. 433.

[6] Educational Testing Service Report, p. 28.

found. That new method, sponsored by the Burroughs Corporation for the teaching and learning of arithmetic with a simple hand operated adding machine with a tape was tried out in the classroom.[7] The responses of the students were noted and studied and the differences were measured along with the results and reactions of other schools in the program.

To say that this new method was or is the one answer to the problem would be presumptuous. The sponsors of the program make no such claims. It was noted, however, that there was a greatly renewed interest in arithmetic and that the average gain in computation and reasoning ability was about twice normal expectancy as measured by standardized tests.

Other than the use of statistics and the classroom observation of the reactions of students there is no yardstick to measure the adequacy or to evaluate the teaching procedures of such a program. It is interesting to note that Ross J. Shaw has listed ten criteria for the evaluation of teaching procedures.[8] The writer believes an affirmative answer can be given to each of these criteria.

[7] *Adventure in Arithmetic*, copyright 1956, Burroughs Corporation.

[8] *N.E.A. Journal*, December 1957, pp. 248 ff. These criteria are reprinted here in outline:

I. Do your procedures, materials, and methods facilitate learning?
II. Will the students work longer and apply themselves more intently?
III. What effect does the new procedure have on the accuracy of the student?
IV. Does the new process bring about a more lasting retention of worthwhile mathematical facts?
V. Is the amount of time always well spent?
VI. Do the methods and materials provide a good understanding of number and number process?
VII. Are the new procedures interesting to the student?
VIII. Do your teaching methods bring students to a higher level of achievement?
IX. Does the procedure cause the student to use his knowledge at home and at school?
X. Does the material used contribute to gradual evaluation into the next number process to be learned?

Any report on a study of the reasons why the new method has renewed interest in arithmetic and has shown such an achievement growth must be recognized as the opinions of the teachers now engaged in teaching such a program. The precepts underlying the method are sound in that they are basic to a much needed background in arithmetic: Understanding of number and process, place value, zero as a placement holder, carrying, borrowing, and the ratio of our decimal number system. Readers will readily agree that these precepts are adaptable to the keyboard of the adding machine. The method, however, allows for much work in estimating, the mental calculations that are verified by machine calculations, the development of fundamentals by cycles, with the resultant recognition of individual differences and individual growth.

Some of the schools that are using this program at the seventh grade level have embarked on a speed-up program which will enable the school to introduce the subject of algebra a year sooner than usual. School administrators are thus taking advantage not only of the achievement gains but of the renewed interest in mathematics.

The interest factor, the writer believes, is a positive approach that should be taken into consideration in any study of the problem before indulging in drawn out discussions as to how and where to increase instruction or revise the curriculum to meet current demands for more and better instruction in arithmetic.

EDITOR'S NOTE. Mr. Taylor spent many years as a teacher of mathematics in Delaware. In seeking a method of improving results he experimented with computing machines and is now associated with the Burroughs Corporation. Currently, there are a number of interesting experiments with computing machines, with manipulative materials, and with reorganization of method and curriculum and all of these are seeking to improve the program and the results in arithmetic. We cannot assess the net results of this experimentation at this time but we should all be interested in seeking to improve the results of instruction in arithmetic. Just what is the role of computing machinery in teaching arithmetic remains a good question. Here too, the method of use as well as the time of use is a critical factor.

Fish and Arithmetic

Dorothy Amsden and Edward Szado
Williamsville, New York

When the North Elementary School in Williamsville recently acquired an acquarium for a second grade classroom, few people realized the effects of this on the total curriculum of the classroom. The aquarium was purchased primarily to bolster the science program. As it turned out perhaps the most marked interests of the students were of a mathematical nature—though they may not have realized it. Becoming cognizant of the opportunity to correlate science and arithmetic we began planning numerous activities which enriched both programs to a marked degree.

Such children's questions as, "How much water will we need?" were soon answered when children attempted to fill the tank by using a gallon jug. This, however, proved to be too heavy for them to lift when full so someone suggested using a milk bottle. But then, how many milk bottles full of water will fill the gallon jug? It soon became evident when we filled the milk bottle with water and poured the water into the gallon jug. Four milk bottles full of water fill a gallon jug (4 quarts in one gallon).

The children then proceeded to fill the tank by using the milk bottle. A child was asked to put a mark on the chalkboard when a milk bottle full of water was poured into the tank. Forty such marks were made. Well then, how many gallon jugs would it have taken? The children proceeded to draw a circle around each four marks. Ten such circles were made. It would take ten gallon jugs to fill the aquarium.

The clerk at the pet shop had said that a pound and a half of sand would be needed for each gallon of water. The problem of, "How much sand to use in the tank?" became the next problem to be worked out. Some children immediately associated the ten gallon capacity of the tank to the amount of sand required. One youngster suggested weighing out one pound of sand for each gallon of water in the tank and then weighing out half a pound for each gallon needed. The total amount of sand weighed was then placed on the scales and the children discovered that ten, one-and-one-half pounds is 15 pounds. These discoveries were printed on a large chart for future reference.

The problem of "How many fish will the aquarium accommodate?" was one that required expert knowledge of aquariums, since both the children and the teacher had no previous experience with aquariums. A hurried telephone call, however, posed another problem for the group, for it was learned that a gallon of water in the aquarium would accommodate an inch of fish, excluding the tail. The children were eager to learn all about inches and measurement of fish. As each fish was brought to class a committee was appointed to measure the fish and record their findings.

Numerous other activities followed these initial experiences in practical arithmetic, feeding the fish $\frac{1}{4}$ teaspoon of food three times a week necessitated the development of a feeding chart. Finding the differences in the temperature in the tank, the room, and the outside air was another outgrowth of this activity.

Of course, science activities were carried on along with the arithmetic, although it may not have followed the traditional concepts taught in the second grade. It is most interesting to observe how much and how well children learn when the learning is necessary for an interesting experience in the classroom.

National Council of Teachers of Mathematics

Report of the Membership Committee

MARY C. ROGERS, CHAIRMAN
Roosevelt Junior High School, Westfield, New Jersey

THE MEMBERSHIP COMMITTEE of the National Council of Teachers of Mathematics is happy to make this report to you, and to congratulate you for your achievement in bringing about the greatest membership total in the history of the Council.

We believe this record of achievement is one of which each of us should be justifiably proud. A very great deal of this success is due you, the current members of the Council. Your increasingly keen enthusiasm and your direct assistance as individuals have been major contributing factors toward the very commendable membership growth evidenced in the factual data of this report.

When Dr. Howard F. Fehr and the Board of Directors of the Council challenged us to an ultimate 25,000 membership, I think many of us really wondered whether such a goal could be reached. We thought we were being quite optimistic in planning to reach that goal in *five years*. It now seems quite possible that it can be reached in *three years or perhaps even less time.*

With current emphasis on the improvement of mathematics education at all levels; with the greatly enhanced interest of the general public in such improvement; with imminent changes in the content and point of view in keeping with Modern Mathematics, it naturally follows that the great majority of mathematics teachers are not only interested in—but feel the need for—closer cooperation with such outstanding educational agencies as the National Council of Teachers of Mathematics.

Now—as never before—*we have an excellent opportunity to acquaint these teachers and their administrators with the fine services of the Council.* This should result in greatly increased membership during the coming months.

Record of Membership Growth

In studying the data relative to membership growth, it might be well to review the goals set up at the Annual Meeting in April 1957 at Philadelphia:

April 1957–April 1958	17,000
April 1958–April 1959	19,000
April 1959–April 1960	21,000
April 1960–April 1961	23,000
April 1961–April 1962	25,000

You will be delighted to know that the membership had already reached a *total of 18,324* when the official count was made on *February 1, 1958*. This total is *108% of the suggested April 1, 1958 goal and 96% of the April 1, 1959 goal*. It indicates a *gain of 3,149 members* since the official count of February 1, 1957. Perhaps we did not "raise our sights" high enough at the Philadelphia meeting.

Recent communications from M. H. Ahrendt, Executive Secretary at our Washington Office, state "The growth and vitality of the Council are increasingly evident in our daily mail.—If history repeats itself and this growth continues throughout the remainder of the school year, *we ought to reach a total of nearly 20,000 by May 1, 1958.*" With your continued support and assistance, we are confident that this high total will be reached as indicated.

In preparing the accompanying analysis, we have indicated State goals originally planned for April 1958 and also similar goals planned for April 1959. We are reporting specific achievements based on both of these goals.

Based on the 17,000 goal originally planned for April 1, 1958, 60% of all states and territories had already reached their goals or gone beyond them when this report went to press on February 15, 1958. These

states and territories are:

1. Arizona	303%
2. Oregon	175%
3. Nevada	167%
4. California	139%
5. Montana	137%
6. Canada	133%
7. Utah	130%
8. New Hampshire	127%
9. South Dakota	127%
10. Wyoming	124%
11. Washington	122%
12. Idaho	120%
13. Oklahoma	120%
14. Connecticut	119%
15. Michigan	118%
16. Florida	117%
17. New Jersey	116%
18. Pennsylvania	114%
19. Maine	113%
20. New York	112%
21. Texas	110%
22. Colorado	109%
23. Hawaii & U. S. Poss.	107%
24. Louisiana	106%
25. Mississippi	106%
26. Illinois	105%
27. Maryland	104%
28. Kentucky	103%
29. Iowa	101%
30. Kansas	101%
31. Ohio	101%

Future Plans

We suggest that future procedures should continue to follow closely those that have proved most effective in the past.

1. We urge a continuance of the "Each One Win One" technique by all present members of the Council.
2. Let us "step up" our publicity and publicity releases in keeping with the rapid strides being made toward the adoption of programs of Modern Mathematics in our schools. Let us acquaint our teachers of mathematics with specific services of National Council in this regard.
   a. We cannot emphasize too strongly the importance of National Council publications, *The Mathematics Teacher*, THE ARITHMETIC TEACHER, *The Mathematics Student Journal*, the many Supplementary Publications, the publications of results of Research in Mathematics Education, the Yearbooks. Currently invaluable is the Twenty-third Yearbook—*Insights into Modern Mathematics*. Attention should be called to the fact that special prices are given to Council members on all Council yearbooks.
   b. Equally valuable to teachers of mathematics will be the recommendations of the National Council Curriculum Committees.
   c. National Council convention programs provide a great deal of highly practical assistance to teachers of mathematics at all levels of education.
3. Your prompt renewal of membership greatly facilitates the keeping of records in the Washington Office and the preparation of reports which are "right up to the minute."
4. At this time particularly, you who are members of the mathematics staff in our State Teachers Colleges and other great Schools of Education, can be of tremendous help in securing members for National Council. This applies to under-graduate students—*potential junior members* of National Council—as well as to your many graduate students most of whom are employed in the field of mathematics education or in the general education of the Elementary School. Students qualify for junior membership throughout the *four years of their under-graduate study*, provided their status as students is endorsed each year with the NCTM Washington by the School of Education which they attend. It should be noted here that "the student membership fee remains the same as now, with one journal available for $1.50 a year and both for $2.50."
5. Directors and staff members at Mathematics Institutes—both Summer and Academic Year —as well as leaders of work-study groups can also be of great assistance in publicizing National Council services and in obtaining memberships.
6. You who are supervisors and/or mathematics department chairmen are obtaining increasingly fine results in your work with your teachers. We are counting on your continued help.
7. State and local associations of mathematics teachers, in cooperation with state representatives to the National Council, are performing an outstanding service to the Council. A very great deal of the strength of the Council is due to your loyalty and your enthusiastic support. A continuance of your fine work is vital to the success of Council services.
8. Library and other institutional subscriptions will continue to be included in preparing reports of membership growth.
9. We believe that each of you understands the necessity for increase in membership fees as explained by Mr. Ahrendt in his article in the February issue of THE ARITHMETIC TEACHER. We sincerely hope—and trust—that the Council will continue to receive the loyal support that has been increasingly manifest among its members and potential members, and that membership totals will continue to grow at an even greater rate than in the past.

The Membership Committee extends to each of you its sincere thanks for your outstanding cooperation and helpfulness. Please accept our best wishes for your greatest professional success at all times.

"1" and "1" is "11"

Frank L. Wolf
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Second and third graders and even older students are intrigued by riddles. We much older folk in our sophistication may not find humor or inspiration in some of the riddles they find amusing, but perhaps we can find enlightenment. Please consider the following examples.

1. Q. What is in the middle of America and also in the middle of Australia?
   A. An R.
2. Q. What is the longest word in the English language?
   A. Smiles. Because there is a mile between the first and last letters. (It is evidently not antidisestablishmentarianism.)
3. Q. The first part of an odd number is removed and it becomes even. What number is it?
   A. Seven. ($even).
4. Q. What is half of 8?
   A. O. It is, in fact, the lower half.
5. Q. What part of London is in France?
   A. The letter N.
6. Q. What is 1 and 1?
   A. 11. (Not 2).

Each of these riddles capitalize on a certain weakness of our spoken language. The weakness is this. In spoken English there is no convenient way to distinguish between a reference to a thing and a reference to a name of that thing.

In written English there is a device for indicating that we are writing about the name of a thing. This device is to put the name in quotation marks. For example, if I write " "Bill" has four letters.", it is clear that I am referring to the word "Bill" and not to Bill himself. If I had written "Bill has four letters.", I evidently would have been asserting something about Bill's possessions (and not the properties of "Bill").

When we verbalize English we usually omit specific mention of the quotation marks. For instance, if I say, "Antidisestablishmentarianism is long" most people will take it that I have said "(The word) "Antidisestablishmentarianism" is long." It should be clear from the written statement that actually my assertion is nonsense since antidisestablishmentarianism is neither long nor short but is rather a kind of doctrine which cannot be said to have length at all.

To return to our riddles, you will note that I have not used any quotation marks in stating them. This is so that the impact (if that is the proper word) that they might have when verbalized would not be diminished. Looking, for example, at number 5 it should be clear that if the answer is to make sense, the riddle should be read,

Q. What part of "London" is in "France"?

Fortunately, when the riddle is read aloud we may (as is the custom) not mention the quotation marks. If we indicate the marks by some such words as "quote" and "unquote", we spoil the riddle. In the answer to number 2, it is simply false that "smiles" has a mile between its first and last letters. It does, however, have "mile" between them and if we verbalize and omit mention of the quotation marks, the answer seems to make sense.

At this point it would be helpful if the reader returned to the given riddles and supplied quotation marks where they will do the most good in clarifying the given questions and answers. (In one case, something more drastic than quotation marks may be required.)

What has all this to do with arithmetic teaching? In the opinion of the author—a great deal. Many things done in arithmetic can be greatly clarified if the (semantic) distinction between a thing and a name of

that thing is kept clearly in mind. After all, it is true that "1" and "1" make "11", whereas "1 and 1 make 2" is also true.

Consider also the following statement which is taken from a certain state instruction guide for the use of arithmetic teachers: "The numerals from one to nine are used to make numbers larger than ten and, when so used, take on a value because of the place in which they appear in the number." What is meant here by "the numerals from one to nine"? Surely the *symbols* (or *names*) "1", "2", "3", "4", "5", "6", "7", "8", "9". (Incidentally what happened to "0"?) But do we make numbers by use of these symbols? Surely not! We use these numerals to make names for numbers. How can a numeral *take on a value?* Is a numeral a variable? To consider it to be one would seem to lead to chaos. How can a numeral *appear in a number?* A number is an abstract concept—it is a property of certain sets. How on earth can a numeral (which is a symbol) *appear* in an abstract concept? Surely this is nonsense. Numerals may, of course, appear in a name of a number, but how they could *appear in the number* is hard to see.

Consider another statement from the same instruction guide: "Fractions must have the same denominator, or name, before they can be added." This one seems to compound confusion with confusion. How can mathematics serve as a model for clarity for the students in the face of such? The statement seems to say that fractions must have the *same name* before they can be added. Does this mean we can add fractions only when they are equal? It seems to, for if two numbers have the same name they should be equal. Else we have one name which names two different numbers and this does not seem proper. A number may have many different names (e.g. "1", "$\frac{3}{3}$", "$2-1$", "1.0", "$\sqrt{1}$" are all names for 1) but to have the same name corresponding to several different numbers would be a disaster. (Unless we wish to use variables as in algebra where a variable is precisely that—a name which may refer to any of a set of numbers.) Even if we strike out the words "or name" in the above given statement from the guide, it is difficult to see how it can make sense. If what is asserted by the statement is true, what shall we do with the student who writes

$$\frac{5}{5}+\frac{7}{7}=2?$$

Quite clearly the student has made a true assertion whether he ever found new names (which had equal denominators) for the fractions or not. Further, the statement seems to say that we cannot add $\frac{1}{2}$ and $\frac{1}{3}$. But it is clear to all my readers that "$\frac{1}{2}+\frac{1}{3}$" is a name (one of the many possible names) for a perfectly good number. If we use lengths of paper representing fractional parts of some basic unit length, there is no doubt that if I lay off a strip of length $\frac{1}{2}$ unit next to a strip of length $\frac{1}{3}$ unit, I find that I have a certain definite combined length. Yet I am told I cannot add $\frac{1}{2}$ and $\frac{1}{3}$. This almost sounds like part of a plot to confuse the student.

What the guide is trying to say (or, at least, what it should be trying to say) is something like this. "$\frac{1}{2}+\frac{1}{3}$" designates (i.e., is a name for) a certain number. In many cases it is not a very convenient name and we would like to find a more convenient name if we can. One way to do this is to find new names for the fractions $\frac{1}{2}$ and $\frac{1}{3}$ which make it easy to find a simple name for $\frac{1}{2}+\frac{1}{3}$. In fact, we see that another name for $\frac{1}{2}$ is "$\frac{3}{6}$". Another name for $\frac{1}{3}$ is "$\frac{2}{6}$". Then

$$\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$$

where the last step may be clearly illustrated with strips of paper marked off in sixths of a unit.

The instruction guide from which the above illustrations were taken was written some years ago and it might be thought that things have improved greatly in more recent times. There are indications that this is not so. Consider the following statement lifted from a recent issue of The Arithmetic Teacher: "The teacher then writes the fraction $\frac{2}{4}$ on the board, and asks the children if they can figure out a way to change this fraction to $\frac{1}{2}$ without using an object or a drawing."

The first thing we notice (and it is perhaps trivial) is that the teacher has done the impossible. You can no more write $\frac{2}{4}$ on the board than you can write President Eisenhower on the board. It is, of course, possible to write "$\frac{2}{4}$" or "President Eisenhower" on the board. Maybe we should accept this slip as a commonly accepted slovenliness of language.

The second thing we notice is, I think, quite serious. Earlier in the article from which the above statement was lifted it was emphasized that the student must already know that $\frac{2}{4}$ *equals, or is the same as,* $\frac{1}{2}$ (which is true). Now the teacher asks the student to change $\frac{2}{4}$ to $\frac{1}{2}$! How do we *change* $\frac{2}{4}$ to $\frac{1}{2}$ when it is already the same as $\frac{1}{2}$? At this stage a bright student may begin to suspect that the teacher is using his own private language where the words change meaning from one minute to the next. There is no dilemma, of course, if a careful distinction is made between $\frac{2}{4}$ and "$\frac{2}{4}$". It is the latter which we wish to *change* and not the former.

There are many other similar examples in recent issues of THE ARITHMETIC TEACHER, but I will not belabor you with more.

Most elementary teachers teach not only mathematics but reading—beginning English if you like. In beginning English as well as in mathematics I think it would be well to emphasize the distinction between things and names of things. Thus, I advocate teaching rudimentary semantics in the primary grades and believe some knowledge of semantics to be indispensable for a good arithmetic teacher.

I would like to close with a riddle that is perhaps more sophisticated than the ones with which we started.

Q. What is wrong with the following reasoning?

Since 5 is a divisor of 10 we know that 5 is a divisor of the numerator of $\frac{10}{15}$. Hence, because $\frac{10}{15}=\frac{2}{3}$ and because we may substitute equals for equals, 5 is a divisor of the numerator of $\frac{2}{3}$. Therefore, 5 is a divisor of 2.

EDITOR'S NOTE. Professor Wolf calls us to task for our colloquial and "loose" use of the English language. The point is well taken. We use the language to convey ideas and many will argue that if the idea is transmitted and received, the language is satisfactory. Sometimes we seem to be required to be less definitive in expressions with young children than we would be with competent adults. Growth in learning is our aim and we must not confuse the issue by either (a) using language at a level too sophisticated for the learner or (b) using language that is not sufficiently precise to convey an accurate impression or language that teaches people to be slovenly in thought and expression. Certainly, a teacher should set a good example in the many things she does with and before children. As Mr. Wolf points out, we face certain technical "naming difficulties" with our mathematical symbolism. Our big job is to invest symbolism with meaning and significance and let us do this with language that is suited to the age-level of the pupils but let us not encourage "mathematical loose talk."

Summer Meeting of The National Council of Teachers of Mathematics, Colorado State College of Education at Greeley, August 19-20, 1958

Plan to attend this meeting and join in the fun and the discussions on arithmetic and other topics in the teaching of mathematics. The people of the area will give you a wondrous welcome and you will thrill to the scenic grandeur of the Rockies. Dr. Forest N. Fisch, Colorado State College of Education, Greeley, is in charge of local arrangements. In the summer an official program of the meeting will be mailed to Council members. Come and bring a friend.

Attitudes Toward Reading and Arithmetic Instruction

Why the Contrast?

Gordon K. Stevenson
Eugene Field Elementary School, San Diego, Calif.

There exists, and has existed for many years, an apparent difference in attitude toward instruction in the fields of reading and arithmetic. Should such differences exist? Is there any scientific evidence that it is justified? Is now a good time to do away with this difference in attitude?

The difference in attitude consists in feeling that it is wise and proper to let children go as fast and as far as they can in developing reading skills and understanding. Contrasted with this, the attitude toward arithmetic instruction seems to be that one is treading on forbidden grounds if one goes beyond the materials outlined for a particular grade level.

In reading, for example, a teacher will work with three groups—a fast group, an average one, and a slower group. If half dozen or more of the fast group pupils exceed their grade level expectancy on a standardized test by two, three, or even four years, she will tell of this with beaming face and considerable enthusiasm because she knows this is approved as an evidence of her effectiveness as a teacher. She will want, and be given, a collegiate dictionary for the use of these outstanding readers, even though this may be only a fourth or fifth grade class. She will be using encyclopedias and an atlas as books of reference for research work.

But, the elementary teacher who exceeds her grade level in arithmetic instruction is often frowned upon. Even to have and to use a junior high school book in math in the elementary school is looked upon as a crime. If a college book in math were found, I suppose that the teacher would simply be executed.

Is there any sound reason for encouraging a teacher to help pupils to go as far beyond their expected grade level in reading as they are capable of going but to take the opposite attitude toward arithmetic instruction? Or, has this attitude grown up only because of the exact nature of arithmetic learnings and the fact that it is relatively easy to determine when advanced instruction has been given a group of pupils? Is this fact, then, used to say—this is my area of instruction and none of you below me on the educational ladder should teach it?

Today we see demands that we more adequately educate large numbers of students in the field of math. Isn't it entirely possible that pupils in the top group in quantitative thinking ability could exceed their fellow pupils by the same margins in this area as the good reader now does in the field of reading? If this is within the realm of possibility, are we not then failing in our obligations if we artificially delay the instruction in math so that the pace is kept to that of the average or middle group of pupils? Isn't it high time that we experiment with a type of program in math that will give the capable student in this field a chance to go ahead more rapidly than the average?

Again we ask, "Why the contrast in attitudes between the two fields? Shouldn't we be consistent and take the same attitude in math as we have so long held in reading?"

Editor's Note. The editor is very concerned about the bright children in our elementary schools. What shall we do with them? Advance them topically in mathematics as the author suggests or have them dig deeper and learn more in the typical subject matter of their grade? Why do we frown upon skipping a grade? Many of us, in "the good old days," skipped one or two grades and seem not to have been harmed permanently thereby. A fine mind coupled with a will to learn can achieve two or three times as much as the normal expectation of a

public school. In foreign countries, we find pupils greatly advanced. What have these foreign pupils lost that is so precious to us? As we reassess our elementary schools with particular attention to the bright children we may well discover that for them "life adjustment" means more and better learning with pupils of their own intellectual level as stimulating peers. Certainly we must be intelligent in looking at standardized testing. A fifth grade youngster who ranks 8.5 is not doing 8th grade work but is doing as well on that particular test as the average pupil in the 8th grade has done. It is suggested that, in schools where it can be arranged, the brighter pupils be given an opportunity to do both (a) dig deeper and (b) advance in the sequence of arithmetic. But let us not construe arithmetic to be restricted to computations. These bright youngsters need to explore ideas, to think, to discover, and to forge ahead by this means rather than by rote learning and memory.

Pupils Make Problems

One of the standard methods of helping pupils to understand problems is to ask them to make problems similar to one written on the blackboard. Occasionally a problem like the following is presented: "My father is 42 years old and my dog is 8. If my dog was a human being he would be 56 years old. How old would my father be if he was a dog? How old would my father and my dog both be if they were both human beings?"

Children's Views

1. The children were given an assignment that called for the reduction of fractions. A newcomer to the class pointed to his first answer and asked, "Shall I bust it down some more?'

2. One boy who had trouble with a new process asked his teacher for help and added, "Please give it to me in slow motion."

Contributed by
HELEN STRUEVE, Pittsburgh

BOOK REVIEW

Improving the Arithmetic Program, Leo J. Brueckner. New York: Appleton-Century-Crofts, Inc., 1957. Paper, vii+120 pp., $1.25.

In the introductory chapter of this monograph Dr. Brueckner states that its major purpose is to assist schools in evaluating and improving their arithmetic programs. He justifies its publication by asserting that keeping the arithmetic program of a school in line with the needs of our changing society and keeping the instructional procedures in line with the findings of research is a constant problem facing those persons who are responsible for guiding and directing the program. He points out that citizens of the community as well as teachers, principals, superintendents, curriculum directors, and specialists should participate in improvement studies which should involve gathering information concerning the needs of the children and the community, analyzing the strengths and weaknesses of the existing program, and outlining immediate and long-range steps for improving the program. A copy of the guide for the study of the arithmetic program developed under the direction of the author in courses and workshops at the University of Minnesota is included. This guide was developed in connection with the activities of the Minnesota State Committee for the Improvement of Education in 1954–55. The major sections of the guide deal with the (1) objectives and outcomes of the arithmetic program, (2) organization and content of the curriculum, (3) instructional practices and procedures, and (4) availability and adequacy of instructional materials. The subsequent chapters of this monograph contain background material as well as specific suggestions for studying, evaluating, and improving the arithmetic program in these areas.

School administrators who are making plans to study the arithmetic programs of their schools will find many suggestions here for organizing and guiding such a study. Classroom teachers and members of the community who are to participate in the study will have a clearer idea of what they are attempting to do if they read this book

first. The emphasis given to the study of the objectives of the instructional program, and the suggestions to reevaluate each objective and estimate the degree to which each is being achieved should deter those administrators and teachers who are inclined to evaluate school programs in piecemeal fashion. The stress that is placed on the need for careful collection and critical appraisal of data should convince those who are inclined to move rapidly that more than a hasty look is necessary before proper steps can be taken to improve a program.

Examination of this monograph should not be limited to those persons who are concerned only with improving the program in arithmetic. The suggestions found here will be of help in the study of any area of the curriculum especially with regard to (1) planning and carrying through a curriculum development program, (2) evaluating appraisal procedures, particularly the standardized testing program, (3) studying and evaluating instructional practices and procedures, and (4) developing a plan for improving instruction.

In the opinion of the reviewer, insufficient attention is given to the role of the consultant and to the amount of time needed to do such a comprehensive study. Too frequently school administrators directing such studies expect the consultant to take the leadership role and fail to give teacher and citizen committees sufficient time to do thoroughly and well what they have been asked to do. The bibliography seems to be lacking in some respects considering the wide-range of background and experience of the people who will be using this monograph. These shortcomings are relatively minor, however, and do not limit the value of this publication to a very measurable extent. The author has a rich background of experience from which he has drawn to make this a practical guide. Professional and lay people earnestly endeavoring to study and improve the arithmetic programs of their schools who use this guide should be able to identify with confidence points at which changes are desirable.

Joyce Benbrook